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ABSTRACT

This guide is designed to assist engineering teachers in developing an understanding of fluid mechanics in their students. The course is designed around a set of nine self-paced learning modules, each of which contains a discussion of the subject matter; incremental objectives; problem index, set and answers; resource materials; and a quiz with suggested responses. Emphasis is placed upon a rational, analytical approach which can have broad applicability to all fields of engineering. Modules include: (1) Fluid Properties; (2) Fluid Statics; (3) Basic Concepts of Fluid Dynamics and the Equation of Continuity; (4) The Energy Equation; (5) The Momentum Equation; (6) Similitude; (7) Viscous Fluid Flow; (8) Viscous Flow Applications; and (9) Compressible Fluid Flow. (CS)

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CLEMSON UNIVERSITY

Environmental Systems Engineering

INSTRUCTOR'S GUIDE
FLUID MECHANICS: A MODULAR APPROACH

JOHN S. COX

1889

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
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INTRODUCTION

Fluid mechanics is one of the engineering sciences that form the basis for all engineering. It is an extension of the basic sequence of statics and/or dynamics of particles as it relates to the behavior of fluids at rest and in motion. The rational laws of those courses are extended to describe the distribution and diffusion of fluid matter and its various properties throughout a system and the force fields which accompany the system. The fundamental laws governing the flux of mass, momentum, and energy from point to point in a fluid system are derived from one or more rational viewpoints, and the resulting general mathematical forms of these principles are applied to a variety of real-world problems. An engineer must be proficient in fluid mechanics since it plays a large role in many aspects of engineering. From the design board and laboratory studies, to the on-site construction of structures, the science of fluid mechanics is important. From studying the motion of blood as a bioengineer, to determining the discharge, sedimentation, and evaporation of a stream as an environmental engineer, the laws of fluid mechanics are vital in engineering analysis. It is important that the engineer should be proficient with the principles of fluid mechanics, and it is the goal of this guide to assist in the development of this proficiency.

HOW TO USE GUIDE

Introduction

By following the procedure presented in this guide, the instructor should be able to guide the student through fluid mechanics in a definitive manner. Once the student finishes the prerequisites, he is given a broad view of the course and what he must learn before completing the course. The fluid mechanics material is divided into modules that stress a single principle or law and its application. By dividing the entire course material into separate modules of study, the student will be more readily able to understand and apply the concepts. Since each module is set up in a standard form, it will become apparent to the student what is expected of him and how the material will be presented.

Prerequisites

Before beginning this course, the student should be able to accomplish the objectives that are listed under the prerequisite heading. The understanding of the concepts presented in these objectives is essential to the understanding of the material in this guide. Therefore, if the student is unable to accomplish these objectives, he should not begin the material on fluid mechanics but should refer to suggested remedial studies.

Course Description

An overall view of the fluid mechanics material in this guide is presented here. This view is only a general description and by no means attempts to define what information is contained in the separate modules.

Terminal Objectives

The terminal objectives explain to the student what the overall objectives are, based upon the incremental objectives from each module. Actually, there is at least one terminal objective for each module stressing what the student should know based upon the information in that particular module. In a sense, the terminal objectives are more generalized compared to the incremental objective. Upon completion of the course, the student should have mastery of the terminal objectives.

Module Components

The modular format provides flexibility to the instructor while providing clarity to the student. Also, information that is formatted as such can be a valuable reference to those who later become involved with fluid mechanics. Each module stresses a particular principle or theory so that the student may work with one or two new ideas at a time and become proficient in using these ideas before going on to the ideas in the next module. The concepts of a particular module are built upon the information in the preceding modules. Each module is divided into several headings that will aid both the instrutor and the student while working with this guide. These headings are as follows:

Subject matter - This heading includes a narrative statement of the subject matter that will be introduced in the module and its practical application in engineering, or possibly its role in future studies. This information informs the student why the material is important and that it is applicable to eingineering work.

Incremental objectives - The objectives state the performance required by the student by the time he is finished with the module. By using these objectives, both the instructor and student agree on what information the student is responsible for. To reinforce this, the module quiz is formulated from these objectives. Thus the objectives serve not only as a guideline for study, but it also aids the instructor in determing quiz problems. The number in parentheses (1.1) represents the objective number. The first digit of the number is the module number, and the second is the objective number of that module.

Problem index - The problem index has been included to precede the problem set. From this index, both the student and instructor can determine what each problem in the problem set pertains to. If a student is weak in a certain area of theory, he can refer to this index as an aid in determining what problems to work or to study more rigorously in order to become more proficient in that area. Also, since this index precedes the problem set, a better relationship between the concept and its application to a problem can be established.

Problem set - The problem set enables the student to put into use the principles and theories introduced in the material. If the student can apply the principles in solving the problems, he has become more knowledgeable of the material in that he can not only recognize the theory, but also put it to use. By providing the problem set, the instructor can determine where the student is weak and what should be stressed more to overcome this weakness. Also, in a sense, the student becomes more confident in working with the principles and theories presented and can see just how they are used in applications. Each problem in the problem set has been coded with a number as indicated in the parentheses (1.1.1:2). The first number is the module number; the second number is the objective number of that module; the third number represents the number of the problem that pertains to the objective; the fourth number is the total number of problems that pertain to that particular objective in the module. Therefore, the next problem of module one that pertains to objective one will be 1.1.2:2. This means that this is the second and last problem that pertains to objective one. There are several advantages in using this particular type of system. First, an orderly manner can be kept with respect to the module and objective numbers. The student can always refer to the sheet of objectives and review the objective that pertains to the problem. Second, by using the last two digits, the student can tell how many problems pertain to that one objective and how many he has already worked. Time does not have to be spent in hunting for problems that will relate the concepts stated in a particular objective. And last, this coding has been formulated to work as a flexible filing system for any problems that are added or taken away. If an instructor wishes to add or subtract problems for an objective, only the last number will change. Also, existing problems can be replaced with new problems without changing the numbers at all, provided they pertain to the same objective.

Therefore, this system is used as a continuous filing, accumulating, and deleting process that will provide flexibility to the instructor and fresh material to the student. These numbers also appear in the master problem set key in the appendix. This can be used for quick guidance for particular topics contained in the entire course.

Problem set answers - By providing answers to the problem set, the instructor does not have to spend time working out answers for the students during class. These problem set answers are given out after the problem set has been handed in and graded (optional). By handing back the problem set answers along with the student's corrected problem sets, the instructor can quickly show the student where the errors are and why. Also, the student will have a collection of problem sets and answers that will be useful in studying for the quiz and for future reference.

Resource material - This will be included in every module so that there will be no confusion as to where the instructional material for module comes from. This material may be in the form of texts, film loops, lecture notes, and other references. Each reference will have a reference number which will be defined in the Master Resource Material List.

Quiz - Problems that are derived from the objectives of the module make up the quiz. This will be used to gage the student's level of comprehension.

Quiz answers - The quiz answers are used to show the student exactly what he did right or wrong. Unless low grades warrant explanations, the answers should save time in class. Also, the collection of quizzes and answers serve as a valuable reference for the student.

Master Problem Set Key

A master key for all of the problems from all the modules within the guide is given to aid both the student and the instructor. Problem numbers are listed under specific headings of theory covered throughout the guide. This index provides a fast way of referring to problems that involve a certain principle or theory.

Master Resource Material List

This list contains the references used throughout the entire guide. Each reference has a number assigned to it that is used in the modules rather than writing down the source every time. In the master resource material list, the number will appear first and then the name of the resource. Other descriptive information follows in the next columns such as page numbers and film loop numbers.

Sample Module

This module is a sample of a self-paced module that consists not only of the module components listed previously, but also of lecture notes combined with the tapes of the lectures covering this particular module. By using a module as such, the student should be able to work through the module at his own pace. At the end of the module, he should be able to complete the module quiz and start on the next module. An entire course set up in this manner can be used with little or no regular class meeting and very little instructor interaction. The ultimate goal of this guide is to develop this course into such a course. Only the tapes and lectures need to be inserted.

Sample Prerequisite Exam

This example should give the instructor an idea of how a prerequisite exam should be set up. The prerequisites listed for this course are used as a basis for formulating the exam. This exam, like other exams, should show the weaknesses of the student and what he should try to review before entering this guide.

PREREQUISITES

Before beginning this course, the student should be able to:

1. Find the roots of a quadratic equation.
2. Solve simultaneous equations containing three variables.
3. Given a quadratic equation, plot the curve on any two dimensional coordinate systems and write the resulting equation if the origin is shifted a number of units in a specified direction.
4. Given a quadratic equation, compute the critical points and plot the curve on rectangular coordinates.
5. Define briefly what a derivative is and give a simple example of the physical interpretation or use.
6. Compute the first derivative of a given function and evaluate it at the indicated point.
7. Compute the derivative of a given function.
8. Examine a function and determine if it is continuous or discontinuous. Also determine if it is differential and if so, evaluate the derivative at a given point.
9. Define a partial derivative.
10. Compute the total derivative using the chain rule when given a functional relationship.
11. Given a Taylor series expansion, expand a specific angle function (such as sin x) about a given point (such as $x_o \frac{\Pi}{4}$).
12. Show by the fundamental theorem of integral calculus that integration can be related to an area under a curve.
13. Integrate a given function without a reference.
14. Integrate a given function by parts.

15. Using boundary conditions and a second order differential equation, derive a specified form of an equation.
16. Define a vector and give an example.
17. Given two vectors, graphically determine their sum.
18. Calculate the scalar and cross product of two vectors in base vector notation.
19. Give a vector associated with a point in space, find the component of that vector in the direction of a specified line at a defined point.
20. Compute by using cylindrical coordinates acceleration of a particle moving in a designated path and with a constant velocity.
21. Draw a free body diagram of a given figure.
22. Given a floor beam supporting a distributed load, compute the following:
    (1) a free body diagram of the beam.
    (2) the equivalent single force which has the same effect on the supports as the original distributed force system.
    (3) the location of this single force so that the moment is the same as the initial force system.
    (4) the forces at each support to maintain the beam in equilibrium.
23. Use force analysis in a pulley system to calculate a designated force.
24. Define the following:
    (1) centroid
    (2) mass center
    (3) center of gravity
25. Compute a designated centroidal coordinate of a given cross-sectional area.

26. Compute the moment of inertia of a given cross-section with respect to a designated axis.

27. Given the intensity of load per unit area, W, on a horizontal plate (varying according to $W = 20 x$ lb/ft^2), compute the magnitude of a single equivalent force which has the same effect as the original loading and locate its position with respect to point O.

28. Calculate the potential and kinetic energy of a body given the weight, acceleration, and height of drop.

29. Compute the velocity, momentum, work, and losses due to friction of an object having a specified weight and moving with a designated force.

30. State the first law of thermodynamics. Write the first law in a mathematical form which is applicable to (a) an open system, (b) a closed system.

31. Compute the internal energy of a fluid given the temperature and pressure.

32. Compute the specific volume for a fluid given the temperature and pressure.

33. State the second law of thermodynamics and give a brief example of an application.

34. Given the thermodynamic processes that a gas is carried through, illustrate the processes on the T-S and P-V diagrams and qualitatively define the heat transfer, work and internal energy changes that will take place.

35. Define the difference between energy in transition(i.e., heat and work) and intrinsic energy (i.e., internal or kinetic energy).

36. Compute the final temperature, the heat flow, and entropy change of a gas during a piston movement within a cylinder. (All information such as pressure, temperature, and work will be provided.)

COURSE DESCRIPTION

The behavior of fluids at rest or in motion, including a study of fluid properties is presented in this course. Emphasis is placed upon a rational, analytical approach from which are developed basic principles of broad applicability to all fields of engineering.

This course has as its basic purpose to develop in the student an appreciation of a fundamental approach to the solution of real-world problems involving fluids in motion and at rest. The fact that most such problems can be formulated using the fundamental conservation laws of mass, momentum, and energy is stressed, and the student is urged to view each solution as a development from these laws rather than resort to end point, specialized equations. The student is also introduced to complementary techniques of analysis based upon the phenomenological theory of the boundary layer and such devices as dimensional analysis and dynamic similarity. Vector mathematics is employed wherever applicable and convenient, particularly in the derivation of the generalized theorems and in the application of the principle of flux of a property through the surface of a control volume. Potential theory of the flow of ideal fluids is presented to point out the rigor and depth of the mathematical model and its applicability to real-world problems.

TERMINAL OBJECTIVES

At the conclusion of this course in Fluid Mechanics, you should be able to:

1.) Obtain the important fluid properties (specific weight, density, viscosity, surface tension, bulk modulus of elasticity, and sonic velocity) for gases and liquids of engineering importance from tables and charts or by mathematical equations.

2.) Compute the magnitude and location of a force exerted by a static fluid on the walls of its container.

3.) Compute the pressure difference between two points which are connected by a manometer containing a suitable indicating fluid.

4.) Apply the conservation of mass principle (continuity equation) to problems of engineering interest in fluid mechanics.

5.) Apply the conservation of energy principle (first law of thermodynamics) to problems of engineering interest in fluid mechanics.

6.) Apply the conservation of momentum principles to problems of engineering interest in fluid mechanics.

7.) Apply dimensional analysis and similarity principles to problems involving models and the generation of dimensionless groups for presentation of experimental data.

8.) Analyze problems (using the basic equations and concepts developed in the course) in the following areas:

   a.) open channel flow
   b.) pipe flow
   c.) boundary layer flow
   d.) flow over bluff bodies

MODULE 1

FLUID PROPERTIES

SUBJECT MATTER

Matter may be classified as either solid, liquid or gas depending on the relative magnitude of intermolecular forces and molecular motion. Since fluid mechanics is not concerned directly with solids, the attention will be focused on liquids and gases.

The difficulties associated with attempts to characterize liquids and gases from a molecular reference frame are obvious. Alternatively, statistically significant groups of molecules of a fluid are found to exhibit characteristics which can be used for its description. These characteristics or properties are very useful for the vast majority of problems encountered in engineering practice and you must become familiar with them. However, whenever the scale of the problem approaches the mean free path of the molecules then fluid properties as usually defined are no longer valid and the characteristics of individual moleculars must be known.

This module covers the important fluid properties which relate to fluid phenomena and the systems of units which are currently in use to express these properties.

INCREMENTAL OBJECTIVES

At the conclusion of this module you should be able to:

1.1) State, from recall, the fundamental and derived units (and any dimensional constants required) for the gravitational, engineering, metric and S. I. systems.

1.2) Calculate (using Newton's law of viscosity) shear stress when the coefficient of viscosity and shear rate are given, coefficient of viscosity when shear stress and shear rate are given, and shear rate when shear stress and coefficient of viscosity are given.

1.3) From recall state the definitions of absolute and kinematic viscosity and locate numerical values for gases and liquids from the charts and tables in Appendix C in Reference #1.

1.4) From recall state the definitions of density, specific weight, specific volume and specific gravity.

1.5) From recall state the perfect gas law and compute

a.) pressure when temperature and density are given

b.) temperature when pressure and density are given

c.) density when pressure and temperature are given

1.6) From recall write the basic definition of bulk modules of elasticity and obtain a numerical value for water from Table C.1 in Appendix C in Reference #1.

1.7) From recall state the basic definition of surface tension and obtain numerical values for water from Table C.1 in Appendix C in Reference #1.

PROBLEM INDEX

Problem Number	Problem Description
1.1.1:1	Unit conversion
1.2.1:3	Rheology
1.2.2:3	Newton's law of viscosity
1.2.3:3	Newton's law of viscosity
1.3.1:1	Kinematic viscosity
1.4.1:1	Density; specific weight; specific gravity
1.5.1:2	Perfect gas law
1.5.2:2	Perfect gas law
1.6.1:1	Bulk modulus of elasticity
1.7.1:1	Surface tension

PROBLEM SET #1

1.1.1:1 Does the weight of a 5-lb. bag of flour denote a force or the mass of the flour? What is the mass of the flour in slugs? What are the mass and weight of the flour at a location where the gravitational acceleration is one-seventh that of the earth's standard?

1.2.1:3 Classify the following substance (maintained at constant temperature):

a.)	du/dy, rad/sec.	0	3	4	6	5	4
	τ, lb./ft.^{2}	2	4	6	8	6	4
b.)	du/dy	0	0.5	1.1	1.8		
	τ	0	2	4	6		
c.)	du/dy	0	0.3	0.6	0.9	1.2	
	τ	0	2	4	6	8	

1.2.2:3 A Newtonian fluid is in the clearance between a shaft and a concentric sleeve. When a force of 100 lb. is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 2 ft./sec. If 300-lb. force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

1.2.3:3 A steel cylinder is 12 in. long and 1 in. in diameter. It falls, because of its own weight, at a uniform rate of 0.5 ft./sec. inside a tube of slightly larger diameter. A castor-oil film of constant thickness is between the cylinder and the tube. Determine the clearance between the cylinder and the tube. Determine the clearance between the tube and the cylinder. The temperature is 100°F.

1.3.1:1 A fluid has a viscosity of 4 centiposes and a density of 50 $lb_m/ft.^3$. Determine its kinematic viscosity in English units and in stokes.

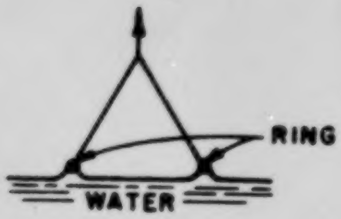
1.4.1:1 The density of a substance is 2.94 gm/cm^3. What is its a.) specific gravity b.) specific volume, and c.) specific weight?

1.5.1:2 A gas at 60°F. and 30 psia has a volume of 4.0 ft ^{3} and a gas constant R = 48 ft -lb /lb $_m$ °R. Determine the density and number of slugs of gas.

1.5.2:2 1.5 lb$_m$ of hydrogen is confined in a volume of 1 ft.^{3} at -40°F. What is the pressure?

1.6.1:1 For K = 300,000 psi for bulk modulus of elasticity of water, what pressure is required to reduce its volume by 0.5 percent?

1.7.1:1 A method of determining the surface tension of a liquid is to find the force needed to pull a platinum wire ring from the surface. Estimate the force necessary to remove a 1" diammeter ring from the surface of water at 68°F. Why is platinum used as the material for the ring?



PROBLEM SET #1--ANSWERS

1.1.1:1 a.) Weight or gravitational attraction denotes a force.

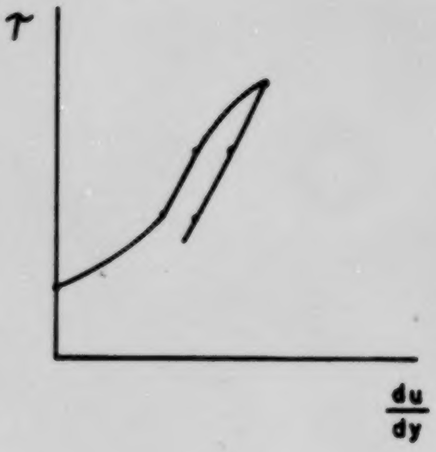
b.) $\text{Mass} = \frac{W}{g} = \frac{5}{32.3}\ \frac{\text{lb.}}{\text{ft./sec.}^2} = 0.155\ \frac{\text{lb-sec.}^2}{\text{ft.}}$ (slugs)

c.) Mass is the same everywhere, 0.155 slugs

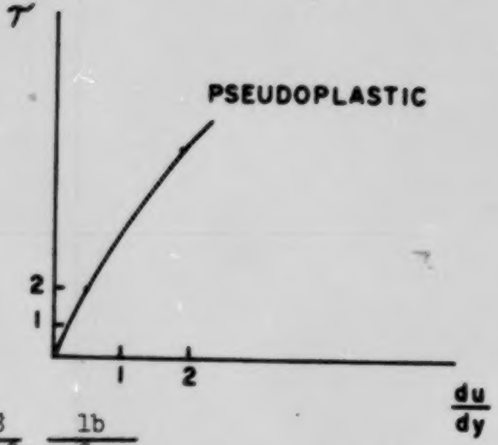
d.) Weight = (0.155) 1/7(32.2) = .713 lb.

1.2.1:3 a.) Non-Newtonian

(note hysteresis)



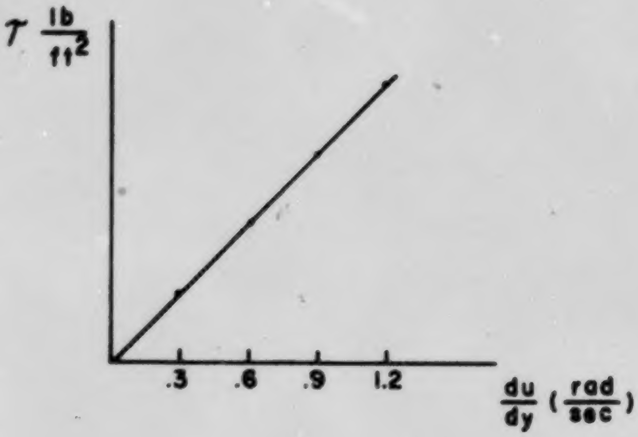
b.) Non-Newtonian



c.) Newtonian

$$\mu = \frac{\tau}{du/dy} = \frac{8}{1.2}\ \frac{\text{lb}}{\text{ft}^2\frac{\text{rad}}{\text{sec}}}$$

$$\mu = 6.67\ \frac{\text{lb-sec}}{\text{ft}^2}$$



1.2.2:3 Force 100 lb.

$V = 2$ ft/sec

assuming a small clearance

$$\tau = \mu \frac{V}{t}$$

$$\frac{F}{A} = \mu \frac{V}{t}$$

$$100 = \left(\frac{\mu A}{t}\right) 2$$

$$100 = K\ 2$$

$$K = 50$$

$$F_2 = KV_2$$

$$300 = (50)\ V_2$$

$$V_2 = \frac{300}{50} = 6 \text{ fps}$$

since increasing the force to 300 lb. does not change anything in the expression for K, it must remain constant.

Note: This solution neglects end effect which would certainly influence the problem. A more refined analysis would have to consider end effects.

1.2.3:3 Steel cylinder 12" long, 1" diameter

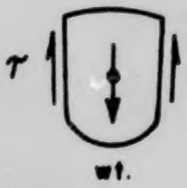
$V_{term} = 1/2$ ft./sec. castor oil film , temp. = 100°F.

(Neglect end effects)

$$\text{wt.} = \gamma \text{ steel } \frac{\Pi(1)^2}{4} (12)$$

$$= 2.83 \frac{\Pi}{4} (12) = 3\Pi\ (.283) \text{ lb.}$$

$\Sigma\ Fy = 0$ Note: buoyant force not included.



$\tau\ (2\Pi\gamma)\ L = Wt.$

But $\tau = \mu \frac{V}{t}$ $\qquad \mu = 6 \times 10^{-3}\ \frac{lb\text{-}sec}{ft^2}$

$\frac{V}{t}\, 6 \times 10^{-3}\, (2\Pi\gamma L) = 3\Pi(.283)$

$\frac{1}{t}\, 1/2\ (6)\ 10^{-3}\ \frac{2\Pi}{12}\, (\frac{1}{R})\ 1 = 3\Pi\ (.283)$

$t = \frac{3(10^{-3})\Pi}{12\ 3\Pi(.283)} = \frac{10^{-3}}{1.2(2.83)}$

$= 2.94 \times 10^{-4}$ft. $\qquad 3.53 \times 10^{-3}$in.

1.3.1:1 $\mu = 4$ centipoise $\qquad \rho = 50\ lb_m/ft^3$, $\qquad \frac{50}{32.174} = 1.554$ slugs

$\mu = .04$ poise , $\qquad \mu = \frac{.04}{479}\,\frac{lb\text{-}sec}{ft^2}$

$\nu = \frac{\mu}{\rho} = \frac{.04(32.2)}{479(50)} = \frac{.4(3.22)}{4.79(5)} \times 10^{-3}$

$= 5.38 \times 10^{-5} ft^2/sec = (30.48)^2 (5.38)\ 10^{-5}$

$\nu = 4998.2 \times 10^{-5}$ stokes $= 4.998 \times 10^{-2}$ stokes

1.4.1:1 $\rho\ 2.94\ g/cm^3$ $\qquad$ dynes $= [\text{gram}\ \frac{cm}{sec^2}]$

a.) sp. gr. = 2.94

b.) $\nu_s = \frac{1}{\rho} = \frac{cm^3}{g} = \frac{1}{2.94} = .340\ \frac{cm^3}{g}$

c.) sp. wt. = $\frac{\text{force}}{\text{volume}}$

$$2.94\ gm/cm^3 = 2.94\ (980.665)$$

$$= 2883.15\ \frac{dynes}{cm^3}$$

check

$$\rho = 2.94\ (62.427) = 183.54\ lb/ft^2$$

$$\gamma = \frac{2883.15}{3.531 \times 10^{-5}} \times 2.248 \times 10^{-6} = 183.5\ lb/ft^3$$

1.5.1:2 Gas @ 60°F. , 30 psia , $V = 4\ ft.^3$, $R = 48\ \frac{ft\text{-}lb}{lb_m{}^oR}$

Density and number of slugs

$60 + 460 = 520^oR$ $\qquad$ $p\ V = nRT$

$$(30)(144)4 = n(48)(32.2)520$$

$$n = \frac{4(144)30}{32.2(48)(520)}\ (slugs)$$

n = .0215 slugs

$$\rho = \frac{n}{V} = \frac{.0215}{4}\ .0057\ \frac{slugs}{ft^3}$$

1.5.2:2 1.5 lb_m of hydrogen $\qquad$ $p\ V = nRT$

$= 1\ ft^3$ $\qquad$ $p(1) = 1.5\ (R)\ 420$ $\qquad$ $\begin{array}{r} 460 \\ -\ 40 \\ \hline +420 \end{array}$

$T = -40^oF.$ $\qquad$ $p(1) = 1.5\ R\ (420)$

pressure = ? $\qquad$ $R = 766$

$$P = \frac{1.5(7.66)(4.20)\ 10^4}{1} = 10^4\ (1.5)(7.66)(4.20) = 48.25 \times 10^4 \text{ psi}$$

$$= \frac{482.5}{144} \times 10^3 \text{psi} = 3.35 \times 10^3 \text{ psia}$$

1.6.1:1 $K = 300{,}000$ psi $K = -\frac{\Delta P}{\frac{\Delta V}{V}}$

$$\frac{\Delta V}{V} = \frac{V_2 - V_1}{V_0} \times 100 = .5\% \quad , \quad \frac{\Delta V}{V} = .005$$

$$300{,}000 = \frac{\Delta P}{(.005)} \quad , \quad \Delta P = .005\ (300{,}000)$$

$$\Delta P = 5(300) = 1500 \text{ lb/in}^2$$

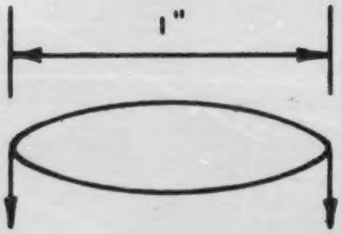
1.7.1:1 Force = $2\Pi r\sigma$ $\sigma = .501 \times 10^{-2}$ lb/ft. (From Table C. 1, Reference #1)

$$\text{Force} = \frac{2\Pi}{12}\ (1/2)(.501)\ 10^{-2} \text{lb.}$$

$$= \frac{\Pi(.501)}{12}\ 10^{-2} = .131 \times 10^{-2} \text{lb.}$$

$$= 1.31 \times 10^{-3} \text{lb.}$$

Platinum because of corrosion resistance



RESOURCE MATERIAL

Ref. No.	Chapter	Pages	Other Description
1	1	3-18	Articles 1.1-1.9
2			Handbook of tables
3			FM-72 Film Loop

QUIZ

1.) A tank of nitrogen is delivered to your laboratory. The delivery man wants to charge you for 1 slug of gas. The gas is at room temperature (70°F.), the pressure gage reads 986 psi, and the barometer in the lab reads 14 psia. The volume of the tank is 2 cu. ft. How many slugs of gas should you pay for?

2.) A mass of 500 Kg weighs how many Newtons under standard gravitational conditions?

3.) Glycerin at 100°F. is being used in a flow system and some fluid properties are needed.

a.) determine the absolute viscosity in lb-sec/ft^2.

b.) determine the kinematic viscosity in ft^2/sec.

c.) determine the density in slugs/ft^3.

4.) For the viscometer shown, the velocity is measured to be

$$u = 3y^2 + 4y + 6 \quad \text{ft/sec.}$$

At the wall (y=0), the shear stress is measured to be

$$4 \times 10^{-3} \frac{\text{lb}}{\text{ft}^2}$$

Compute the coefficient of viscosity in $\frac{\text{lb-sec}}{\text{ft}^2}$

QUIZ ANSWERS

1.) $pV = nRT$

$$n = \frac{1000\ (144)2}{(55.2)(32.2)530} = \frac{2000(144)}{55.2(32.2)(530)} = .3057$$

$n = 306$ slugs

2.) $N = Kg \times a$ $\qquad 32.2\ \frac{ft}{sec^2} \times 12\ \frac{in}{ft} \times 2.54\ \frac{cm}{in}$

$N = 500\ (9.81)$

$N = 4907$ Newtons

3.) a.) $\mu = 3.9 \times 10^{-3}\ \frac{lb\text{-}sec}{ft^2}$

b.) $\nu = 1.45 \times 10^{-3}\ ft^2/sec.$

c.) $\nu = \frac{\mu}{\rho} \qquad \rho = \frac{\mu}{\nu} = \frac{3.9 \times 10^{-3}}{1.45 \times 10^{-3}} = 2.69\ \frac{slugs}{ft^3}$

4.) $\mu = 3y^2 + 4y + 6$

$\tau = \mu\frac{du}{dy}\ \Big|y = 0$

$\tau = \mu(4)$

$4 \times 10^{-3} = \mu\ (4)$

$\mu = 1 \times 10^{-3}\ \frac{lb\text{-}sec}{ft^2}$

MODULE 2

FLUID STATICS

SUBJECT MATTER

Now that we have investigated the important properties that characterize both liquids and gases and have looked at particular systems of units to express these properties, our study of fluid mechanics can continue.

Although the most common problem to confront the engineer will involve fluids in motion, there are a sufficient number of applications which involve fluids at rest to justify our consideration. Manometry, forces on submerged plane surfaces and Archimedies' Principle are but a few of the applications which will be discussed in this module. Fluids in motion will be pursued in Module 3.

INCREMENTAL OBJECTIVES

At the conclusion of this module you should be able to:

2.1 From recall, state the basic equation of fluid statics and apply it to fluids of constant density or fluids of variable density.

2.2 Given a manometer with the lengths and specific weights of all fluid columns known, determine the pressure difference $P_A - P_B$ for any two designated points A and B.

2.3 For a submerged inclined plane surface of known geometry, identify the free surface and use this together with the equations

$$F = h_c A$$

$$Y_{cp} = Y_c + \frac{I_c}{AY_c}$$

$$X_{cp} = X_c + \frac{I_{xy}}{AY_c}$$

to obtain the resultant force and center of pressure.

2.4 For a submerged curved surface of known geometry, identify the free surface and use the equations for a plane surface shown in Objective 3 to computer the magnitude and center of pressure of the resultant force.

2.5 From recall, state the two forms of the principle of buoyancy (Archimedies' Principle). Express both in mathematical form and compute buoyant forces and their respective points of application.

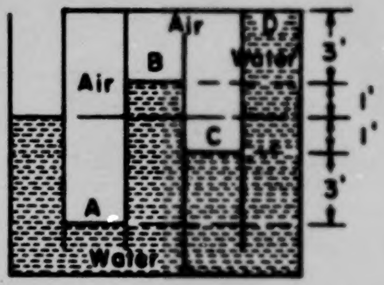
2.6 For a fluid moving as a rigid body with constant acceleration (linear or normal), compute the pressure at any point from the known pressure at a given point.

PROBLEM INDEX

Problem Number	Problem Description
2.1.1:3	Pressure variation in a static liquid
2.1.2:3	Pressure variation in a static gas
2.1.3:3	Pressure variation in a static liquid of variable density
2.2.1:4	Manometry
2.2.2:4	Manometry
2.2.3:4	Manometry
2.2.4:4	Manometry
2.3.1:5	Forces on submerged plane surfaces
2.3.2:5	Forces on submerged plane surfaces
2.3.3:5	Forces on submerged plane surfaces
2.3.4:5	Forces on submerged plane surfaces and location of center of pressure
2.3.5:5	Forces on submerged plane surfaces and location of center of pressure
2.4.1:3	Forces on submerged curved surfaces
2.4.2:3	Forces on submerged curved surfaces
2.4.3:3	Forces on submerged curved surfaces and location of center of pressure
2.5.1:2	Archimedies Principle (Buoyancy)
2.5.2:2	Archimedies Principle (Buoyancy)
2.6.1:4	Relative equilibrium (acceleration of a fluid mass with no relative motion)
2.6.2:4	Relative equilibrium
2.6.3:4	Relative equilibrium

PROBLEM SET #2

2.1.1:3 The container of the figure below holds water and air as shown. What is the pressure at A, B, C, and D in pounds per square foot?

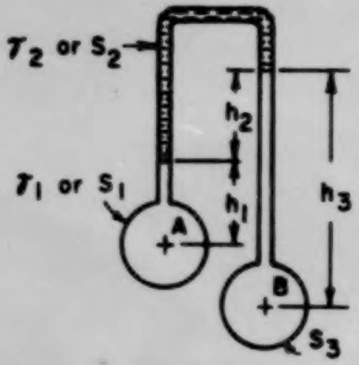


2.1.2:3 For isothermal air at 20°F, determine the pressure and density at 10,000 ft. when the pressure is 15 psia at sea level.

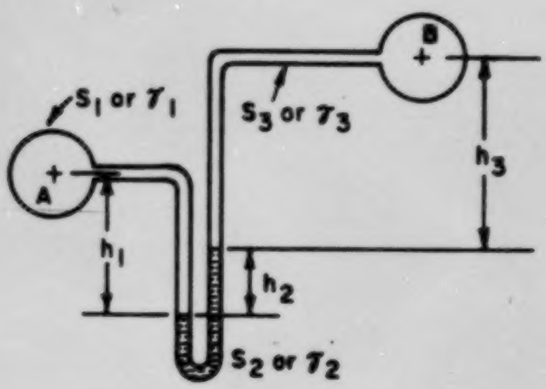
2.1.3:3 What is the pressure at a point 50 ft. below the free surface in a fluid that has a variable density in slugs given by $\rho = 1.9 + ah$, in which $a = 0.01$ slugs/ft^4 and h is the distance in feet measured from the free surface?

2.2.1:4 Express a pressure of 8 psi in: (a) inches of mercury, (b) feet of water, (c) feet of acetylene tetrabromide, sp. gr. 2.94.

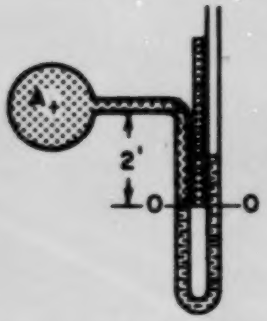
2.2.2:4 In the figure below, $S_1 = 1.0$, $S_2 = 0.95$, $S_3 = 1.0$, $h_1 = h_2 = 1.0$ ft and $h_3 = 3.0$ ft. Compute $P_A - P_B$ in inches of water. (S = sp.gr. = specific gravity)



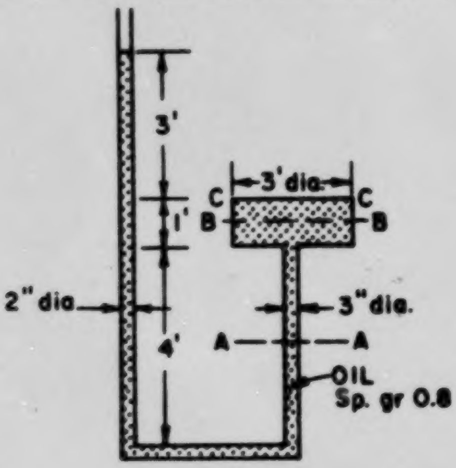
2.2.3:4 In the figure below, $S_1 = S_3 = 0.83$, $S_2 = 13.6$, $h_1 = 16$ in., $h_2 = 8$ in., and $h_3 = 12$ in. (a) Find P_A if $P_B = 10$ psi. (b) For $P_A = 20$ psia and a barometer reading of 29.0 in. find P_B in feet of water gage.


B
S_1 or γ_1
S_3 or γ_3
A
h_3
h_1
h_2
S_2 or γ_2

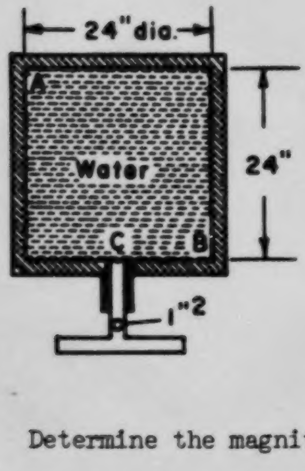
2.2.4:4 In the figure below, A contains water and the manometer fluid has a specific gravity of 2.94. When the left meniscus is at zero on the scale, P_A = 3 in. water. Find the reading of the right meniscus for P_A = 1 psi with no adjustment of the U-tube scale.

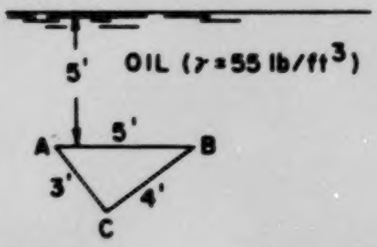


2.3.1:5 Find the force of oil on the top surface CD of the figure below if the liquid level in the open pipe is reduced by 4.0 ft.


3'
3' dia.
C
C
1'
B
B
2" dia
3" dia.
4'
A
A
OIL
Sp. gr 0.8

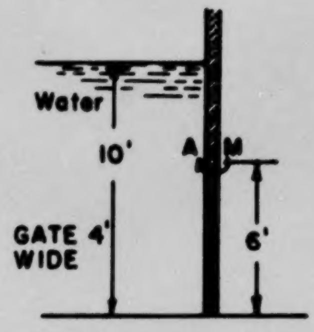
2.3.2:5 The cylindrical container of the figure below weighs 100 lb. when empty. It is filled with water and supported on the piston. What force is exerted on the upper end of the cylinder? If an additional 150-lb. weight is placed on the cylinder, how much would the water force against the top of the cylinder be increased?


24" dia.
A
Water
C
B
24"
1"2

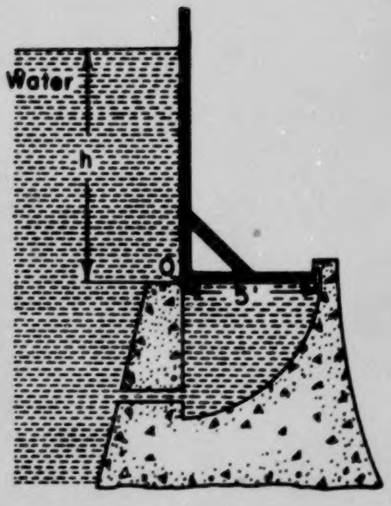

5'
OIL (γ=55 lb/ft³)
5'
A
B
3'
4'
C

2.3.3:5 Determine the magnitude of the force acting on triangle ABC of the above figure : (a) by integration and (b) by formula.

2.3.4:5 If there is water on the other side of the gate (figure below) up to A, determine the resultant force due to water on both sides of the gate, including its line of action.



2.3.5:5 The gate of the figure below weights 300 lb/ft normal to the paper. Its center of gravity is 1.5 ft from the left face and 2.0 ft above the lower face. It is hinged at O. Find h for the gate just to come up to the vertical position shown.

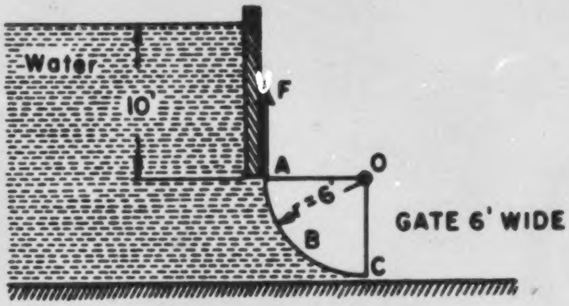


2.4.1:3 (a) Determine the horizontal component of force acting on the radial gate (figure below) and its line of action.

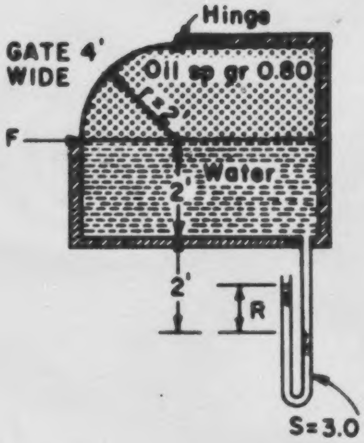
(b) Determine the vertical component of force and its line of action.

(c) What force F is required to open the gate, neglecting its weight?

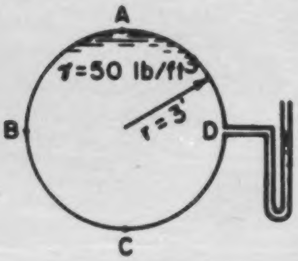
(d) What is the moment about an axis normal to the paper and through point O?


Water
10'
F
A
O
r=6'
B
C
GATE 6' WIDE

2.4.2:3 Calculate the force F required to open or hold closed the gate of the figure below when R = 1.5 ft.


Hinge
GATE 4' WIDE
Oil sp gr 0.80
r=2'
F
Water
2'
2'
R
S=3.0

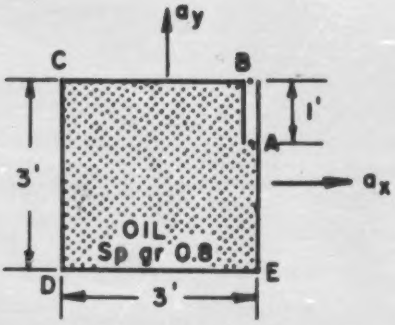
2.4.3:3 The cylinder (figure below) is filled with liquid as shown. Find (a) the horizontal component of force on AB per foot of length, including its line of action, and (b) the vertical component of force on AB per foot of length, including its line of action.



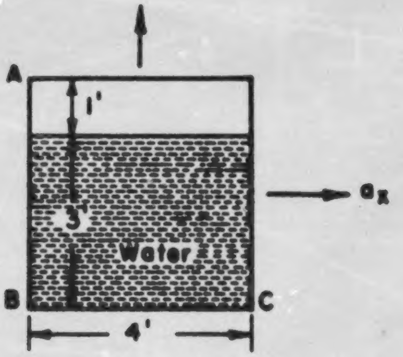
2.5.1:2 Two cubes, of the same size, 27 ft^3, one of sp. gr. 0.80, the other of sp. gr. 1.1, are connected by a short wire and placed in water. What portion of the lighter cube is above the water surface, and what is the tension in the wire?

2.5.2:2 How many pounds of concrete, $\gamma = 150\ lb/ft^3$, must be attached to a beam having a volume of 4 ft^3 and sp. gr. 0.65 to cause both to sink in water?

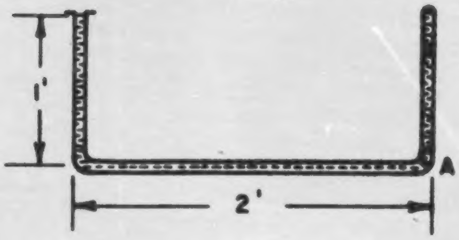
2.6.1:4 In the figure below, $a_x = 8.05\ ft/sec^2$, $a_y = 0$. Find the imaginary free liquid surface and the pressure at B, C, D, and E.



2.6.2:4 In the figure below, $a_x = 16.1$ ft/sec^2, $a_y = 16.1$ ft/sec^2. Find the pressure at A, B, and C.



2.6.3:4 The tube (figure below) is filled with liquid, sp. gr. 2.40. When accelerated to the right 8.05 ft/sec^2, draw the imaginary free surface and determine the pressure at A. For $P_A = 8$ psi vacuum determine a_x.



2.6.4:4 A vessel containing liquid, sp. gr. 1.2, is rotated about a vertical axis. The pressure at one point 2 ft. radially from the axis is the same as at another point 4 ft. from the axis and with elevation 2 ft. higher. Calculate the rotational speed.

PROBLEM SET #2--ANSWERS

2.1.1:3 $P_1 = 0$ psig

$$P_A = 4(62.4)\ \text{lb/ft}^2 = 250\ \text{lb/ft}^2$$

$$P_B = P_A - 5(62.4) = -62.4\ \text{lb/ft}^2$$

$$P_C = P_B + 2(0.075) = -62.25\ \text{lb/ft}^2$$

$$P_D = P_C - 5(62.4) = -62.25 - 5(62.4) = -374\ \text{lb/ft}^2$$

All gage pressure

2.1.2:3 Isothermal Air @ 20^oF

$$P = P_o \rho \left(- \frac{y - y_o}{P_o/g\rho_o}\right)$$

here $P_o = 15$ psia $y_o = 0$

$$P = 15\rho \left(- \frac{10,000}{P_o/g\rho_o}\right)$$

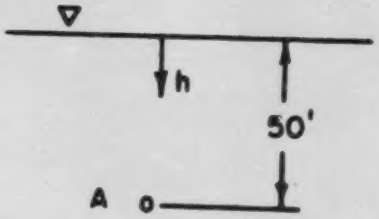
$$= \frac{15(144)}{(32.2)\ \frac{15(144)}{(53.3)g(480)}} = (53.3)(480)$$

$$P = 15\ \rho - \frac{10,000}{53.3(480)} = 10.15\ \text{psia}$$

$$\frac{P_o}{\rho_o g} = \frac{RT_o}{g}$$

$$\rho_o = \frac{P_o}{RT_o}$$

substituting, $\rho_o = .00178\ \text{slugs/ft}^3$



2.1.3:3 Surface pressure = 0 psig

$$\frac{dp}{dh} = \gamma$$

$$\gamma = \rho g = (1.9 + (0.01)h)g$$

$$\int_0^P dp = \int_0^{50} (1.9g + (0.01)hg)dh$$

$$(P_{50} - 0) = 1.9\,gh\,\Big]_0^{50} + (0.01)g\frac{h^2}{2}\Big]_0^{50}$$

$$P_{50} = 1.9(32.17)50 + \frac{(32.17)(0.01)}{2}(50)(50)$$

$$= 3056 + 402.2$$

$$= 3458\ \text{lb/ft}^2$$

2.2.1:4 Pressure of 8 psi

(a) $$\text{P psi } (144) = 8_{Hg}\frac{h_{in}}{12} = 13.55(62.4)\frac{h}{12}$$

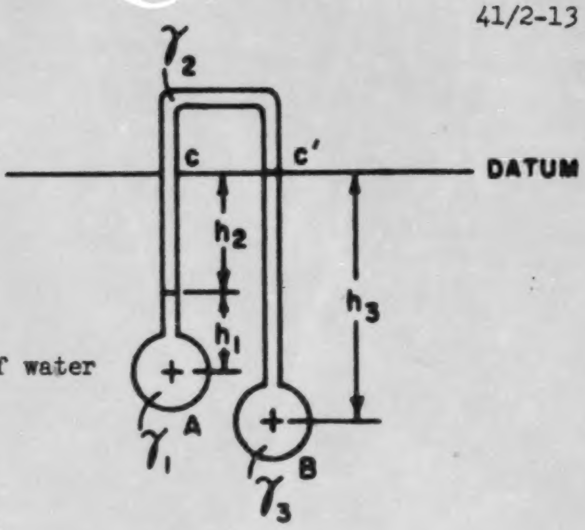
$$h_{in} = \frac{8(144)(12)}{13.55(62.3)} = 16.4 \text{ in. Hg}$$

(b) $$h_{\text{in water}} = \frac{8(144)(12)}{(1)(62.3)} = 221.9 \text{ in. water}$$

$$h_{ft} = 18.5 \quad \text{(for 8 psi)}$$

(c) $$h_{in} = \frac{8(144)(12)}{(1.94)(62.3)} = 75.4 \text{ in.}$$

$$h_{ft} = \frac{8(144)}{(1.94)(62.3)} = 6.29 \text{ ft. (for 8 psi)}$$



2.2.2:4 $S_1 = 1.0$ $h_1 = 1$ ft

$S_2 = 0.95$ $h_2 = 1$ ft

$S_3 = 1$ $h_3 = 3$ ft

Compute $P_A - P_B$ in inches of water

$P_C = P_{C'}$

$P_A = \gamma_1 h_1 - \gamma_2 h_2 = P_B - h_3 \gamma_3$

But $\gamma_1 = S_1 \gamma_{water} = \gamma_{water} = 62.3$ lb/ft^3

$\gamma_2 = 0.95(62.3)$

$\gamma_3 = \gamma_1 = 62.3$

$P_A - 62.3(1) - 0.95(62.3)1 = P_B - 62.3(3)$

$$\frac{P_A - P_B}{62.3} = -3 + 0.95 + 1 = -1.05$$

$P_A - P_B = -1.05\ (62.3) =$ lb/ft^2

$= -65.4$ lb/ft^2

In terms of inches of water

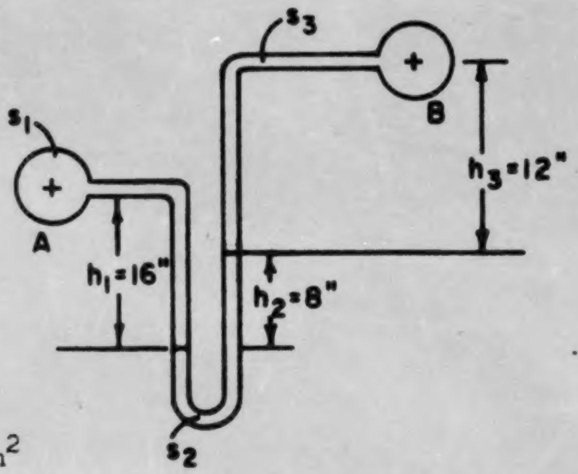
$P_A - P_B = -65.4$ lb/ft$^2 = \gamma_w h_w$

$$h = -\frac{65.4}{62.3} = -1.05 \text{ ft}$$

$h = -12 \times 1.05 = -12.6$ in. water gage

2.2.3:4 $S_1 = S_3 = 0.83$

$S_2 = 13.6$



(a) $P_B = 10 \text{ lb/in}^2$

$$P_A + \frac{16}{12} S_1 \gamma_w = P_B + \frac{12}{12}(S_3)\gamma_w + \frac{8}{12} S_2 \gamma_w \text{ lb/ft}^2$$

$$P_A + \frac{4}{3}(0.83)(62.4) = 10(144) + (0.83)(62.4) + \frac{2}{3}(13.6)(62.4)$$

$$P_A = 1440 - \frac{1}{3}(0.83)(62.4 + \frac{2}{3}(13.6)(62.4)$$

$$= 1440 - 17.26 + 565.8$$

$$P_A = 1988.5 \text{ lb/ft}^2$$

$$P_A = \frac{1988.5}{144} = 13.81 \text{ lb/in}^2 \text{ gage}$$

(b) If $P_A = 20 \text{ lb/in}^2$ absolute

Barometer = 29 in. Hg

$$P_{Barom} = (13.6)(62.4)\frac{29}{12} = 14.24 \text{ lb/in}^2$$

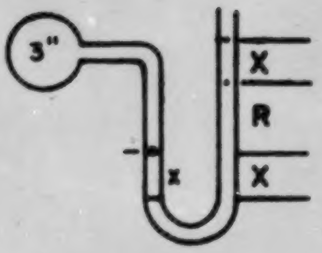
$$P_{A \text{ gage}} = 20 - 14.19 - 5.76 \text{ psig or } 829.4 \text{ psf}$$

$$836.7 + \frac{4}{3}(0.83)62.4 = P_B + 0.83(62.4) + 565.8$$

$$P_B = 829.4 + 17.26 - 565.8 \text{ lb/ft}^2$$

$$= 280.9 \text{ lb/ft}^2$$

$$h_B = \frac{288.1}{\gamma_{water}} = 4.62 \text{ ft water gage}$$



2.2.4:4 sp. gr. = 2.94

3 in. water

3/12 = 0.25 ft water

$$P_o = (2.25)62.4 \text{ lb/ft}^2) = R(2.94)(62.4)$$

$$R = \frac{2.25}{2.94} \times 12 = 9.18 \text{ in.}$$

$$1 \text{ psi} \times 144 + (2 + x)\ 62.4 = (R + 2x)2.94(62.4)$$

$$\frac{144}{62.4} + (2 + x)\ 62.4 = (R + 2x)2.94(62.4)$$

$$2.31 + 2 + x = (\frac{9.18}{12})(2.94) + 2.94(2x)$$

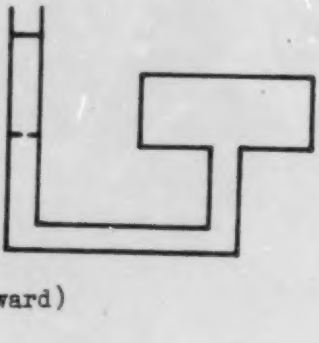
$$4.31 + x = \frac{26.99}{12} + 5.88x$$

$$4.31 = 2.25 + 4.88x$$

$$x = \frac{2.06}{2.88} = 0.422 \text{ ft}$$

Reading = x + R = 5.06 + 9.18 = 14.2 in. = 1.18 ft.

2.3.1:5 How is it reduced? If it is simply removed so that a free surface still remains:

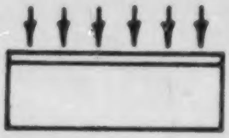


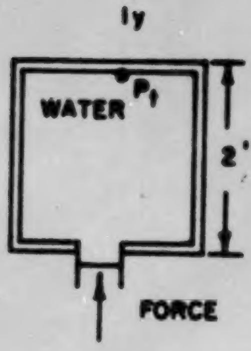
$$P_{CD} = -1(0.8)(62.4)$$

$$\text{Force} = -1(0.8)(62.4)\pi(1.5)^2$$

$$= -(0.8)(62.4)\pi(2.25) \text{ lb}$$

Force = 352.9 lb (tension or directed inward)





2.3.2:5 Tank weighs 100 lb when empty

$\Sigma F_y = 0$

$F = P_{BOT}A = W_{Tank} + W_{Fluid}$

$P_{BOT} = \frac{100 + 62.4\pi(1)^2 2}{1} = 492 \text{ lb/in}^2$

Pressure at the top: $P_T = 492 - \frac{62.4(2)}{144}$

$P_T = 491.13 \text{ lb/in}^2$

Force at the top: $F_T = P_T \text{ Area} = 491.13(\pi)(12)^2$

$F_T = 222,182 \text{ lb}$

Adding 150#

Pressure at bottom: $P_{BOT} = \frac{250 + 62.4(\pi)(1)^2 2}{1}$

$= 642 \text{ lb/in}^2$

Pressure at top: $P_T = 642 - \frac{2(62.4)}{144} = 641.13$ psig

Force at top: $F_T = 641.13\ (12)^2 = 290,042$ lb.

Difference = 290,042 - 222,182 = 67,860 lb

2.3.3:5 (a) By formula:

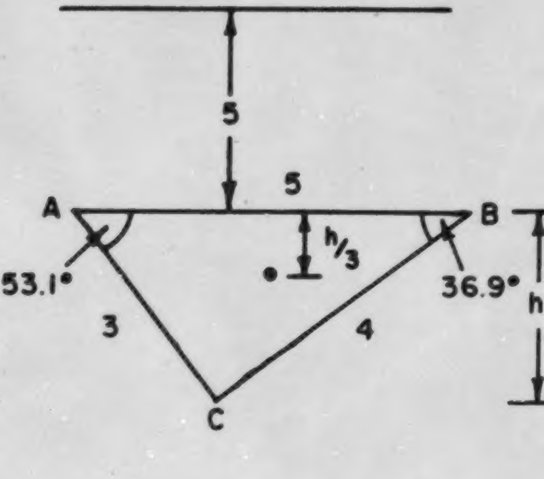
$F = \gamma\bar{h}A$

$\bar{h} = 5 + \frac{h}{3} = 5 + \frac{2.4}{3} = 5.8$ ft.

$h = 4 \sin 36.9^\circ = 3 \sin 53.1^\circ$

$F = (55)(5.8)(6) \rightarrow \frac{\text{lb}}{\text{ft}^3} \times \text{ft} \times \text{ft}^2$

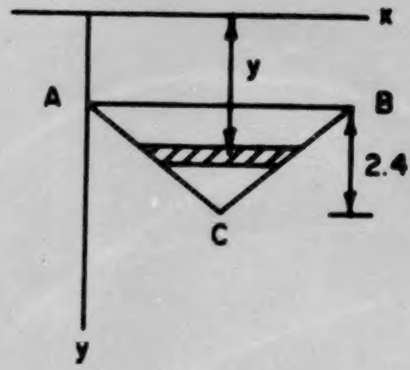
$F = 1914$ lb



45/2-17

(b) By integration

Define coordinate system and write equation for lines AC & BC



AC: $\frac{dy}{dx} = 1.3319$

$y = 1.3319x + C$ when $x = 0,\ y = 5$
$C = 5$

$y = 1.3319x + 5$

BC: $\frac{dy}{dx} = -.7508$

$y = -.7508x + A$ when $x = 5,\ y = 5$
$A = 8.754$

$y = -.7508x = 8.754$

$$dF = PdA = \gamma y dA = \gamma y\ [x_{BC} - x_{AC}]\ dy = \gamma y\ [\frac{(8.754-y)}{.7508} - \frac{(y-5)}{1.3319}]\ dy$$

$$F = \gamma \int_{5}^{7.4} y[11.66 - 1.3319\text{[illegible]}) - (.7508y) + 3.754]\ dy$$

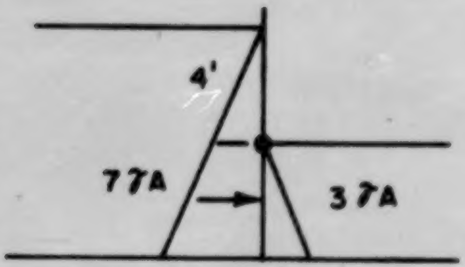
$$= \gamma \int_{5}^{7.4} y[15.41 - 2.083y]\ dy = \gamma \int_{5}^{7.4} 15.41y - 2.083y^2)\ dy$$

$$= \gamma\ [15.41\frac{y^2}{2} - \frac{2.083}{3}y^3]_{5}^{7.4} = \gamma\ \frac{15.41}{2}(7.4^2 - 5^2) - \frac{2.083}{3}(7.4^3 - 5^3)$$

$$= \gamma\ [229.30 - 194.57] = \gamma\ [34.73] = 1910 \text{ lb.}$$

Note: The small difference between the results of (a) & (b) is due to round off.

2.3.4:5



$F_R = 4(62.4)$ Area

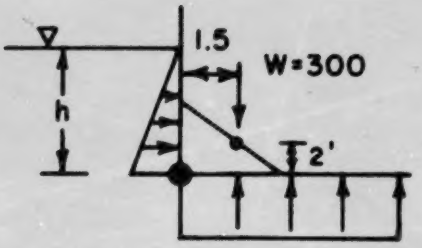
$F_R = [7\gamma - 3\gamma]$ Area

$F_R = 4\gamma(24)$ lb

center of pressure located at the centroid of the area.

Note: This is true only because the same liquid occurs on both sides of the gate. For any other situation, center of pressure and centroid of the area do not coincide!

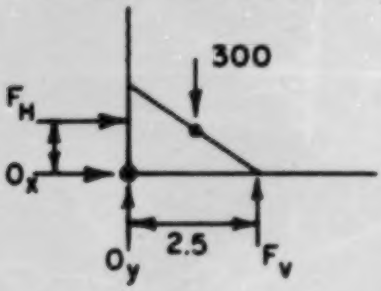
2.3.5:5



ASSUME A GATE WIDTH

From Hydrostatics

FREE BODY DIAGRAM OF GATE



47/2-19

$$F_H = \gamma \bar{h} A = \gamma \frac{h}{2}(h) = \gamma \frac{h^2}{2}$$

$$F_V = \gamma h(5)(1) = 5\gamma h$$

Locate moment arm (From 0 - F_H)

$$Y_{cp} = Y_c + \frac{I_c}{AY_c} = \frac{h}{2} + \frac{1(h)^3}{12(h)(1)(h)/2} = \frac{h}{2} + \frac{h}{6} = \frac{2}{3}h$$

or from 0 to F_H is h/3

$$\Sigma M_0 = 0 \qquad F_H\left(\frac{h}{3}\right) + 300(1.5) - F_V\left(\frac{5}{2}\right) = 0$$

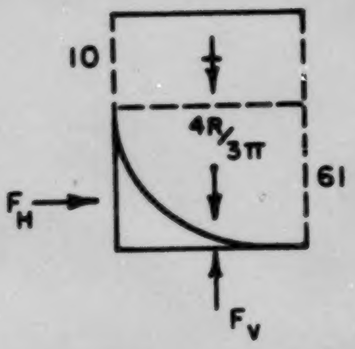
$$\frac{\gamma h^2}{2}\frac{h}{3} + 300(1.5) - 5\gamma h\left(\frac{5}{2}\right) = 0$$

$$\frac{h^3}{6}(1) + \frac{450}{(62.4)} - 12.5(1)h = 0$$

$$h^3 + 43.27 - 75h = 0$$

From which h = 0.58ft.

2.4.1:3



a) $F_H = \gamma \bar{h} A$

$= 62.4(13)(36)$

$= \gamma(13)(36) = 468\ \gamma$ lb

$Y_{cp} = Y_c + \frac{I_c}{AY_c}$

$= \frac{\frac{6(6)^3}{2(12)(6)}}{(6)^2(13)} + 13 = \frac{3}{13} + 13$

$= 13.23$ ft.

b) $F_V = \frac{1}{4}\Pi(6)^2(6)(\gamma) + 10(6)(6)\gamma$

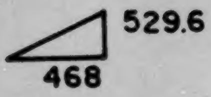
Note: This is the weight of a column of water above curve ABC extending to the free surface.

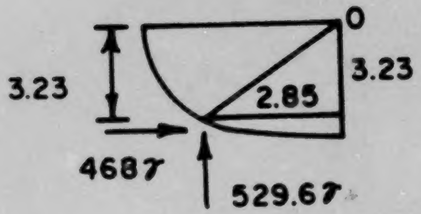
$= \frac{\Pi}{4}(36)(6)\gamma + 360\gamma$

$= (54\Pi\gamma + 360\gamma) = 169.6\gamma + 360\gamma = 529.6\gamma_w$

$$\Sigma M = F_V x \; ; \quad 360\gamma(3) + 54\Pi\gamma\frac{8}{\Pi} = 529.6\ \gamma$$

$$\frac{360(3) + 54(8)}{529.6} = \bar{x}\ , \ \bar{x} = 2.85$$




O
3.23
3.23
2.85
468γ
529.6γ

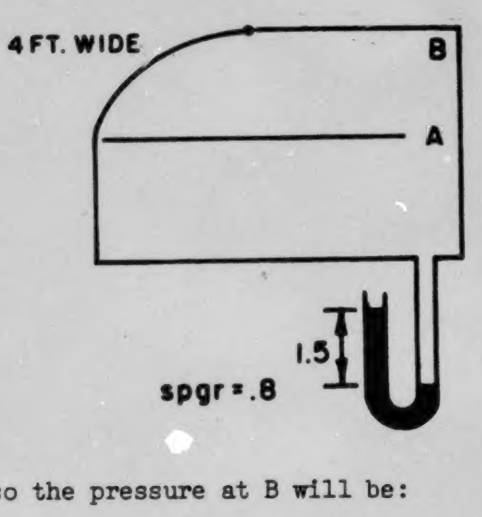
$$\text{slope} = 1.13 = \frac{3.23}{2.95}$$

$$1.13 = \frac{F_V}{F_H}$$

c) Force goes there; therefore moment is zero.

d) Takes a force F only great enough to overcome bearing moment due to friction.

2.4.2:3


4 FT. WIDE
B
A
1.5
spgr = .8

$R = 1.5'$ $\quad P_1 = 1.5(3\gamma_w)$

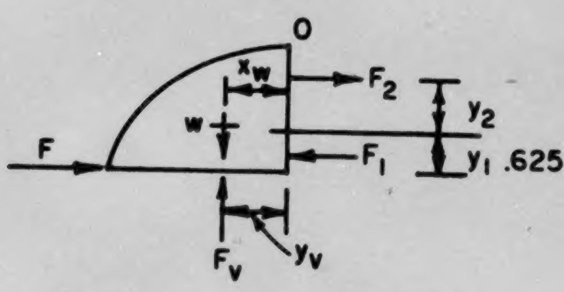
$\qquad = 4.5\ \gamma_w$

Pressure $P_A = 0.5\gamma_w$ lb/ft^2

so the pressure at B will be:

$$P_B = P_A - \gamma_o(2) = 0.5\gamma_w - 2(.8)\gamma_w$$

$$P_B = 0.5\gamma_w - 1.6\gamma_w = -1.1\gamma_w \text{ (vacuum)}$$


O
x_w
F_2
y_2
w
F
F_1
y_1 .625
F_V
y_V

The free surface (P = 0) occurs .625 ft. above the water surface.

Using that free surface, draw the following FBD (free body diagram).

$$F_2 = \gamma_o \bar{h} A = .8(62.4)\left(\frac{1.375}{2}\right)(4)(1.375) = 188.8 \text{ lb.}$$

$$y_2 = {}^2/_3(1.375) = .917 \text{ ft.}$$

$$F_1 = (.8)62.4\left(\frac{.625}{2}\right)4(.625) = 39 \text{ lb.}$$

$$y_1 = {}^2/_3(.625) = .417 \text{ ft.}$$

$$\text{Wt.} = \frac{\Pi(2)^2}{4}(4)(62.4)(.8) = 627 \text{ lb.}$$

$$X_w = \frac{4R}{3\Pi} = .85 \text{ ft.}$$

$$F_V = (.625)(.8)(62.4)(8) = 249.6 \text{ lb.}$$

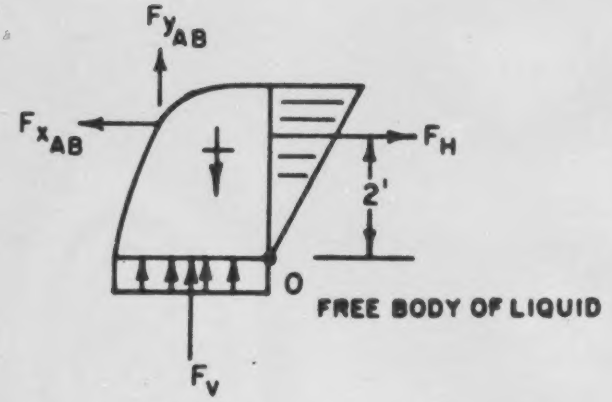
$$X_{FV} = 1 \text{ ft.}$$

$$\Sigma M_O = 0 \qquad F_1(1.792) + F_V(1) - F_2(.458) - W(X_w) - F(2) = 0$$

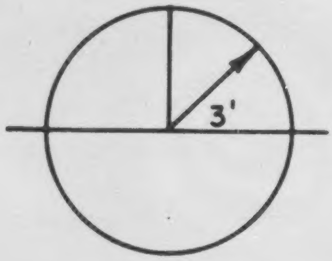
$$16.3 + 249.6 - 86.5 - 53.3 = 2F$$

$$F = -150 \text{ lb.}$$

2.4.3:3



$\gamma = 50 \ {}^{lb}/_{ft^3}$



51/2-23

Horizontal Component (Assume 1 ft. depth)

$F_H = \gamma\bar{h}A = \gamma(^3/_2)(3 \times 1) = 4.5(50) = 225^{\#}$

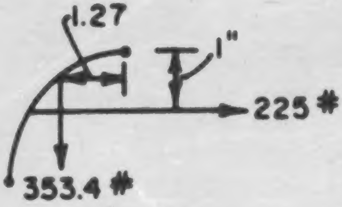
Line of action $Y_{cp} = 2$ ft. (same as force of tank on liquid F_{XAB})

Vertical Force

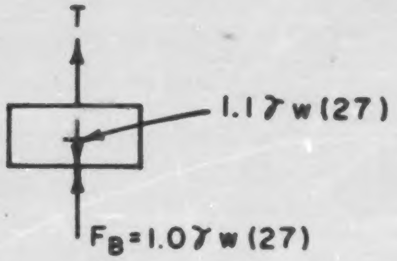
$F_V = \gamma\bar{h}A = \gamma(0)A = 0$

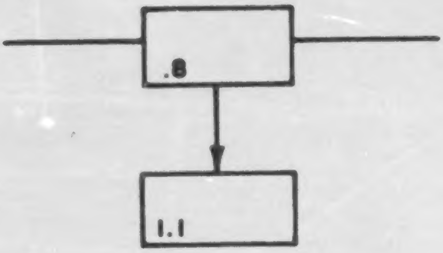
weight $= \frac{\Pi(3)^2}{4}(1)50 = \frac{9\Pi}{4}(50) = 353.4$ lb.

location $\frac{4R}{3\Pi} = \frac{4}{\Pi} = 1.27$ ft. forces on AB



2.5.1:2



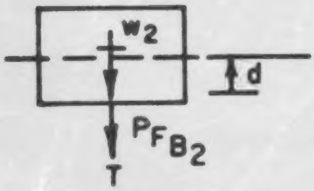


$\Sigma F_y = 0$

$T - 27\gamma_w(1.1) + 27(1)\gamma_w = 0$

$T = 27\gamma_w(1.1-1) = .1(27)\gamma_w = 168.5^{\#}$

FB #2



$\Sigma F_y = 0$

$W_2 + T - F_{B2} = 0$

$F_{B2} = T + W_2 = (.1)(27)\gamma_w + (.8)(27)\gamma_w$

$= \gamma_w V_D$

$V_D = 27(.1) + .8\ (27)$

$= 27(.8 + .1) = .9(27)$

$$V_D = 24.3 \text{ ft.}^3$$

$$3 \times 3 \times 3 = 27$$

$$3 \times 3 \times d = 24.3$$

$$d = \frac{24.3}{9} = 2.7 \text{ ft.}$$

above surface = .3 ft.

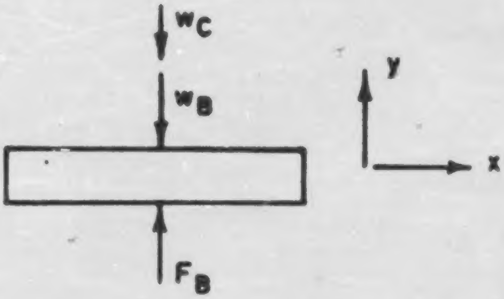
2.5.2:2 Beam volume = 4 ft.^{3} Concrete = γ = 150 lb./$_{\text{ft.}}{}^3$

Beam sp. gr. = .65

When the weight of beam and concrete equals the buoyant force (based on total beam volume) then impending sinking will result.

$$W_{Beam} = V_B \gamma_{Beam}$$

$$W_c = V_c \gamma_c$$



Buoyant force = $F_B = V_D \gamma_{water}$

$$\Sigma F_y = 0$$

$$F_B - W_{Beam} - W_c = 0$$

$$V_b \gamma_w = V_B \gamma + V_c \gamma_c$$

$$(V_B + V_c)\gamma_w = V_B V_B + V_c \gamma_c$$

$$(4 + V_c)62.3 = 4(.65)(62.3) + V_c 150$$

$$4(62.3) - 4(.65)(62.3) = V_c\ [150 - 62.3]$$

$$4(62.3)(1 - .65) = V_c(87.7)$$

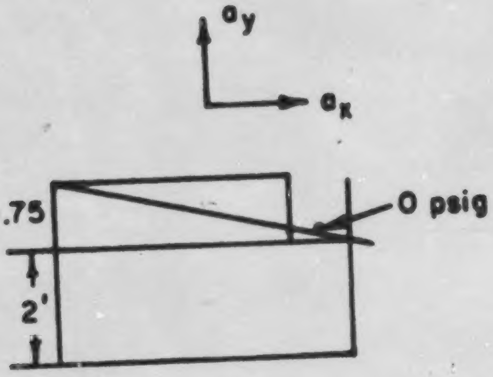
$$87.22 = V_c 87.7$$

$$V_c = .9945 \text{ ft.}^3$$

$$W_c = .9945 \times 150 = 149.18 \text{ lb.}$$

2.6.1:4 $a_x = 8.05 \text{ ft/sec.}^2$

$$a_y = 0$$

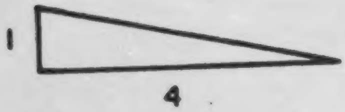


$$\frac{\partial p}{\partial y} = -\gamma\frac{a_x}{g}$$

$$\frac{\partial p}{\partial y} = -\gamma\left(\frac{a_y}{g} + 1\right)$$

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy$$

$$\frac{dy}{dx} = -\frac{a_x}{a_y + g} = -\frac{8.05}{32.2} = -\frac{1}{4}$$



$$P_B = P_A = 0$$

$$P_c = -\frac{1}{4}(62.4)(.8) = -12.5 \text{ psf}$$

$$P_d = 2\frac{3}{4}(62.4)(.8) = +137.3 \text{ psf}$$

$$P_e = 2(62.4)(.8) = +99.8 \text{ psf}$$

$$\frac{\partial p}{\partial x} = -\frac{a_x}{g}\gamma$$

$$\Delta P = -\frac{a_x}{g}\gamma\Delta x \qquad \text{or } P_D - \gamma\left(\frac{a_x}{g}\right)(3)$$

$$= 2.75(.8)(62.4) - (.8)(62.4)\frac{3}{4}$$

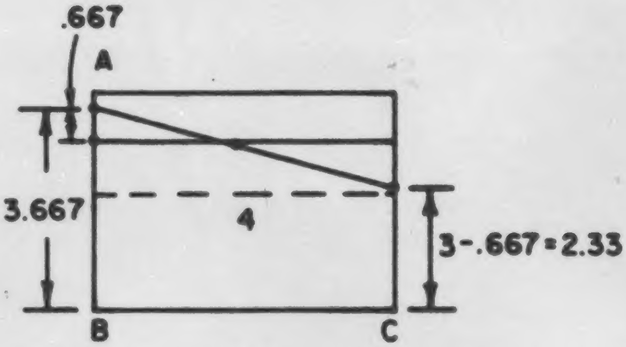
$$= 2(62.4) \text{ psf}$$

2.6.2:4 $a_x = 16.1 \quad a_y = +16.1$

Assume open container

$$\frac{dy}{dx} = -\frac{a_x}{a_y + g} = -\frac{g/2}{\frac{g}{2} + g}$$

$$= -\frac{\frac{1}{2}}{\frac{1}{2} + 1} = -\frac{1}{3}$$



$$\frac{\Delta y}{2} = \frac{1}{3}$$

$$\Delta y = \frac{2}{3}$$

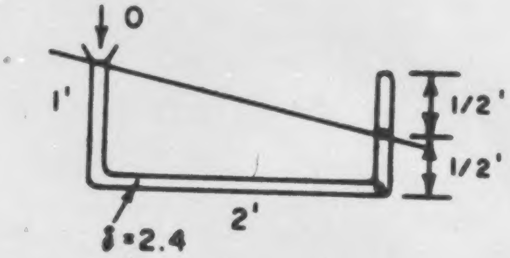
$P_A = 0$ psig ; $P_B = 3.667 \times \gamma(1 + \frac{1}{2}) = 3.667(1.5\gamma)$

$P_B = 2.38$ psig ; $P_C = 2.333\ 1.5\gamma = 1.51$ psig or

$$P_C = P_B - \frac{\gamma}{g}a_x(\Delta x) = 2.38 - \frac{62.4}{32.2}(16.1)(4) = 1.51 \text{ psig}$$

2.6.3:4 $a_x = \frac{g}{4}$

$$\text{slope} = -\frac{a_x}{a_y + g} = -\frac{1}{4}$$



$$P_A = \frac{1}{2}(2.4)\left(\frac{62.4}{144}\right)\text{psig}$$

$$= \frac{1.2(62.4)}{144} = 0.52 \text{ psig}$$

If $P_A = 8$ psi vacuum

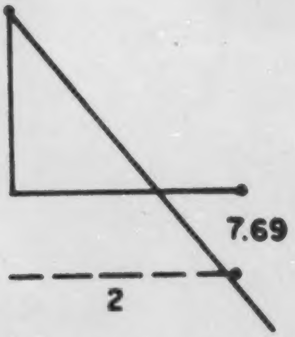
$$8 = h(2.4)\frac{62.4}{144}$$

$$h = \frac{8(144)}{2.4(62.4)} = 7.69 \text{ ft.}$$

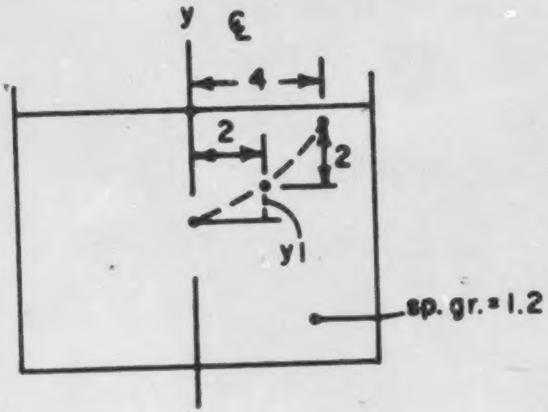
$$\text{slope} = \frac{7.69 + 1}{2} = -\frac{8.69}{2} = -\frac{a_x}{g}$$

$$a_x = \frac{8.69}{2} g$$

$$= 4.35g = 140 \text{ ft./ sec}^2$$



2.6.4:4



$$y = \frac{r^2 \omega^2}{2g}$$

$$y_1 = \frac{4\omega^2}{2g}$$

$$y_2 = (y_1 + 2) = \frac{16\omega^2}{2g}$$

solving:

$$\left(\frac{4\omega^2}{2g} + 2\right) = \frac{16\omega^2}{2g}$$

$$2 = \frac{12\omega^2}{2g}$$

$$\omega^2 = \frac{4g}{12} \quad , \quad \omega = \sqrt{\frac{g}{3}}$$

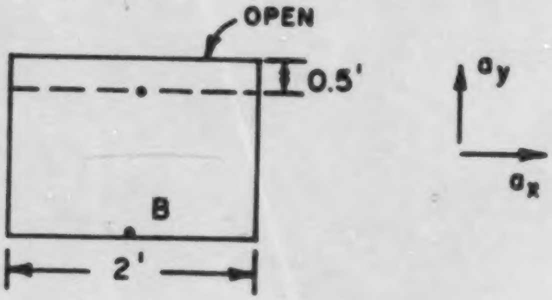
$$\omega = 3.28 \text{ rad./sec.}$$

RESOURCE MATERIAL

Ref. No.	Chapter	Pages	Other Description
1	2	27-78	Articles 2.1-2.9

QUIZ

1. A cubical box 2 ft on each side is filled with water to within 0.5 ft of the top. The box is then accelerated upward at 32.2 fps^2 and to the right at 32.3 fps^2 as shown in the figure. What volume of water, if any, will spill out? What is the pressure at point B?

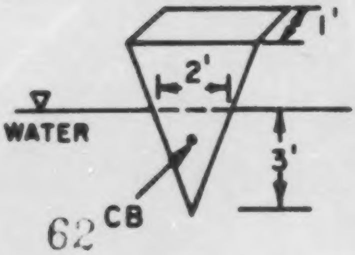


2. State the principle of buoyancy: (a) for a submerged body and (b) for a floating body.

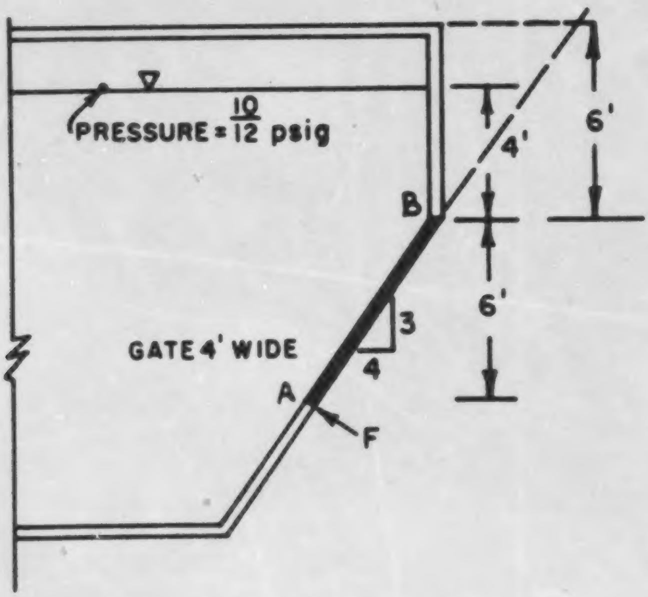
(a)

(b)

(c)
For the body shown below, compute the buoyant force and show it acting at the center of buoyancy.



3. Compute the force which must exist at A for the flow gate AB to maintain its position. The fluid has a specific weight of 60 lb/ft^3.


PRESSURE = $\frac{10}{12}$ psig
6'
4'
B
6'
3
4
GATE 4' WIDE
A
F

QUIZ ANSWERS

1. (a) slope $= \frac{dy}{dx} = -\frac{a_x}{g + a_y}$

slope $= -\frac{g}{2g} = -\frac{1}{2}$

no water spills

(b) $\frac{\partial p}{\partial y} = -\gamma(1 + \frac{a_y}{g}) = -\gamma(1 + 1) = -2\gamma$

$P = 0 + 2\gamma_y = 2\ (1.5) = 187.2$ psfg

2. (a) A submerged body is acted upon by a buoyant force which equals the specific weight of the fluid and the total volume of the body. This force acts through the centroid of the submerged volume.

(b) A body displaces its weight in fluid in which it floats. The buoyant force acts through the centroid of the displaced volume.

(c) $F_B = \gamma V_D$

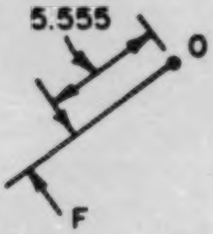
$= 62.4 \times (\frac{6}{2} \times 1)$

$= 62.4 \times 3 = 187.2$ #

60/2-32

3. Free surface

$\frac{10}{12}(144) = 60h \qquad h = \frac{10(144)}{60(12)} = \frac{120}{60} = 2'$



Force = $\gamma \bar{h} A$

$= 60(9)(40)$

$= 21,600$ #

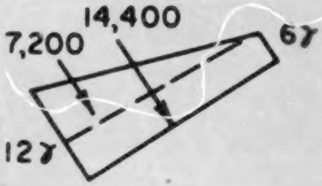
$$y_{CP} = y_C + \frac{I}{A_{YC}} = 15 + \frac{4(10)^3}{12(40)(15)} = 15 + \frac{4000}{12(40)(15)} \equiv 15.555 \text{ ft}$$

$\Sigma M_o = 0$

$F(10) - 21,600(5.555) = 0$

$F = \frac{21,600}{10} \times 5.555 = 12,000$ lb

F = 12,000 # directed



MODULE 3

BASIC CONCEPTS OF FLUID DYNAMICS AND THE EQUATION OF CONTINUITY

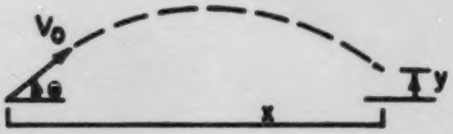
SUBJECT MATTER

In the first two modules of Fluid Mechanics, we have been concerned with fluid properties and the basic analysis of fluid statics. You will certainly agree that the more general problems in fluid mechanics involve fluid motion which is what we will discuss in this module.

Fluid motion involves a different reference frame than you are familiar with. In statics, dynamics and mechanics of materials you have used a fixed coordinate system and specified the motion or deformation with respect to that coordinate system. This technique is called a Lagrangian or Systems method of description. Recall the equations which predict the position of a particle which has an intial velocity V_0 at an angle θ (see sketch).

$$x = V_{0_x} t$$

$$y = -\frac{gt^2}{2} + V_{0_y} t$$



The equations are written as functions of time only. Velocity and acceleration can be obtained simply by taking suitable total derivatives.

Imagine a river flowing beneath a bridge upon which you are standing. Or perhaps the wind on a gusty day. Can you easily follow a particular fluid particle or determine its velocity or acceleration? The fact that you cannot is the reason we need an alternative method of describing the fluid motion. Rather than the Lagrangian formulation, we will use the Eulerian method of description. This enables us to determine the velocity, pressure, acceleration, etc. at a particular point in space, regardless of the particle that happens to occupy that point when we make the observation. Article 3.1 in Ref. #1 develops the basic equation for the rate of change of a property using the Eulerian or Control Volume concept. By properly identifying N as the fluid mass, the conservation of mass or continuity equation can be formulated.

The remainder of this module involves the kinematics of fluid motion. The concepts of streamlines, steady flow, uniform flow and turbulence are defined and will be used in the problem selection.

INCREMENTAL OBJECTIVES

At the conclusion of this module you should be able to:

3.1) From recall, distinguish between the Eulerian or Control Volume and Lagrangian or Systems method of description of fluid motion.

3.2) From recall, derive equation 3.1.6 in reference number one and make suitable definitions of terms to obtain the equation of continuity.

3.3) Reduce the general continuity equation to differential form for use with steady or unsteady, compressible or incompressible flows.

3.4) Reduce the general continuity equation to an integral form for steady duct flow of an incompressible or compressible fluid.

3.5) Given the velocity components in a two dimensional incompressible flow, derive and plot equations for the streamlines.

3.6) Define the following terms (as they pertain to fluid mechanics):

laminar
turbulent
steady
uniform flow
multi-dimensional flow
ideal fluid

PROBLEM INDEX

Problem Number	Problem Description
3.1.1:1	Eulerian and Lagrangian description
3.3.1:2	Differential continuity equation (steady flow)
3.3.2:2	Differential continuity equation (steady flow)
3.4.1:3	Continuity equation for steady pipe flow
3.4.2:3	Integral continuity equation (steady flow)
3.4.3:3	Integral continuity equation
3.5.1:2	Streamline equations
3.5.2:2	Differential continuity equation and streamline equations
3.6.1:1	Power losses in turbulent flow

PROBLEM SET #3

3.1.1:1 Classify the following velocity fields as either Lagrangian or Eulerian:

a) $u = 0.2(x + y) + 3t$

$v = 0.1(x + y) + \frac{1}{2}t$

b) $u = a \sin \Pi(x - ct)$

c) $u = 0.3t^2 + 2t + 5$

$v = 5t$

3.3.1:2 Does the velocity distribution, $\bar{q} = \bar{I}(5x) + \bar{J}(5y) + \bar{K}(-10Z)$ satisfy the law of mass conservation for incompressible flow?

3.3.2:2 Show that the velocity $\bar{q} = \bar{i}\,\frac{4x}{x^2 + y^2} + \bar{j}\,\frac{4y}{x^2 + y^2}$ satisfies continuity at every point except the origin.

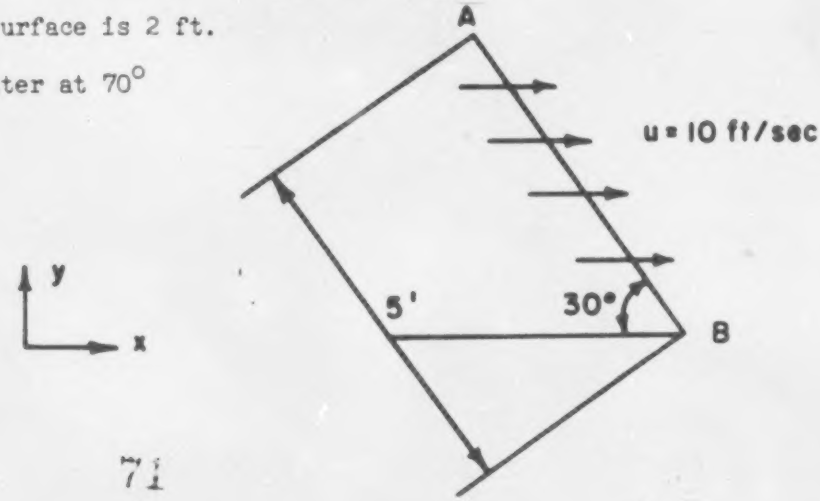
3.4.1:3 A pipeline carries oil, sp. gr. 0.86, at V = 6 ft./sec. in an 8 in. ID pipe. Find the velocity at this section and the mass rate of flow in slugs per second.

3.4.2:3 $\bar{q} = \bar{i}\,\frac{4x}{x^2 + y^2} + \bar{j}\,\frac{4y}{x^2 + y^2}$ is a velocity distribution that is everywhere radial from the origin with magnitude $v_r = \frac{4}{r}$. Show that the flow through each circle concentric with the origin (per foot in the Z direction) is the same.

3.4.3:3 The exit boundary AB of a control volume is shown in the figure below. Compute the mass rate of flow crossing the boundary.

Note: control surface is 2 ft.

$\rho = 1.94$ for water at 70°



3.5.1:2 A three dimensional velocity distribution is given by $u = -x$, $v = 2y$, $w = 5 - Z$. Find the equation of the streamline through (2,1,1).

3.5.2:2 A two dimensional flow can be described by $u = -\frac{y}{b^2}$, $v = \frac{x}{a^2}$. Verify that this is the flow of an incompressible fluid and that the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a streamline.

3.6.1:1 In a flow of liquid through a pipeline the losses are 3 hp. for average velocity of 6 ft./sec. and 6 hp. for 9 ft./sec. What is the nature of the flow?

PROBLEM SET #3--ANSWERS

3.1.1:1 a.) Eulerian

b.) Eulerian

c.) Lagrangian

3.3.1:2 $\bar{q} = 5x\bar{I} + 5y\bar{J} + \bar{K}(-10Z)$

$\bar{\nabla} \cdot \bar{q} = ?$

$$\frac{\partial u}{\partial x} = +5$$

$$\frac{\partial v}{\partial y} = +5$$

$$\frac{\partial w}{\partial Z} = -10$$

$$\Sigma \frac{\partial u_1}{\partial y_j} = 0$$, yes

3.3.2:2 $\bar{q} = \bar{I}\,\dfrac{4x}{x^2 + y^2} + \bar{J}\,\dfrac{4y}{x^2 + y^2}$

for the steady, incompressible flow, continuity is satisfied when

$$\bar{\nabla} \cdot \bar{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = \frac{4x}{x^2 + y^2} \qquad v = \frac{4y}{x^2 + y^2}$$

$$\frac{\partial u}{\partial x} = (x^2 + y^2)\frac{4}{(x^2 + y^2)^2} - 4x(2x) = \frac{4x^2 + 4y^2 - 8x^2}{(x^2 + y^2)^2} = \frac{4y^2 - 4x^2}{(x^2 + y^2)^2}$$

$$\frac{\partial v}{\partial y} = (x^2 + y^2)\frac{4}{(x^2 + y^2)^2} - 4y(2y) = \frac{4x^2 + 4y^2 - 8y^2}{(x^2 + y^2)^2} = \frac{4x^2 - 4y^2}{(x^2 + y^2)^2}$$

adding $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ $\therefore$ continuity is satisfied except when $x = y = 0$

3.4.1:3 oil sp. gr. = 186 , $V = 6$ fps. , 8" ID

$P_2 = 10$ in.

$V_1 A_1 = V_2 A_2$

$$6\left(\frac{\Pi}{4}\right)\left(\frac{8^2}{144}\right) = V_2 \cdot \frac{\Pi}{4}\left(\frac{10^2}{144}\right)$$

$$6(8)^2 = V_2(10)^2$$

$$V_2 = 6\left(\frac{8}{10}\right)^2 = (.8)^2 6 = 3.84 \text{ ft/sec}$$

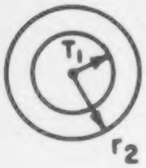
Mass flow $\rho AV = 6\frac{\Pi}{4}\left(\frac{8}{10}\right)^2(.86)(1.94) = 3.5$ slugs/sec.

3.4.2:3 $v_r = \frac{4}{r}$

for flow over area 1

$$v_{r_1} \text{Area}_1 = v_{r_1} 2\Pi r_1(1)$$

$$= \frac{4}{r_1} 2\Pi r_1$$



$$= 8\Pi \text{ ft.}^3/\text{sec.}$$

for flow over area 2

$$v_{r_2} \text{Area}_2 = \frac{4}{r_2} 2\Pi r_2(1)$$

$$= 8\Pi \text{ ft.}^3/\text{sec.}$$

Note: circumferential area is

$2\Pi r Z$ where $Z = 1'$

3.4.3:3 $\int \bar{u} \cdot d\bar{A} = (10\bar{i}) \cdot (\bar{n}dA)$

$\bar{n} = \sin 30\bar{i} + \cos 30\bar{j}$

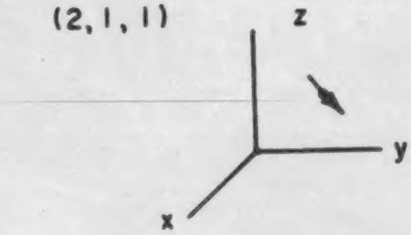
$\int 10\bar{i} \cdot (\sin 30\bar{i} + \cos 30\bar{j})dx$

$\int 10 \sin 30 \, dA = 10 \sin 30 \, (5 \times Z)$

$= 100 \sin 30 = 50$ ft.^{3}/sec.

mass rate of flow $\bar{n} = 50\rho$ for water @ 70°, $\rho = 1.94$

$\bar{n} = 50(1.94) = 97$ slugs/sec.



3.5.1:2 3-D velocity distribution

$u = -x$

$v = 2y$

$w = 5 - z$

$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

$\frac{-dx}{x} = \frac{dy}{zy}$

$-2 \ln x = \ln y + \ln C$

$\ln x^2 + \ln y = \ln C$

xy plane $(x^2y = C)$

$\frac{-dx}{x} = \frac{dz}{(5-z)}$

$-\ln x = -\ln(5-z) + \ln C_1$

$\ln x - \ln(5-z) = \ln C_1$

$\frac{x}{(5-z))} = C_1$

$\frac{x}{5-z} = C_1$

$\frac{dy}{v} = \frac{dz}{w}$

$\frac{dy}{2y} = \frac{dz}{5-z}$

$\frac{1}{2}\ln y = -\ln(5-z) + \ln C_2$

$\ln y = -2 \ln(5-z) + \ln C_2$

$\ln y = \ln(5-z)^2 = \ln C_2$

$y(5-z)^2 = C_2$

$x^2y = C, \ \frac{x}{5-z} = C_1, \ y(5-z)^2 = C_2$

$C = 4$

$x\sqrt{y} = 2$

$y(5-z)\,\frac{x}{C_1} = C_2$

$\frac{(5-z)}{x} = z$

$C_1 = \frac{1}{2}$

$C_2 = 16$

$y(5-z)^2 = 16$

$y\, 4x^2 = 16$

$x^2y = 4$

$x\sqrt{y} =$

$x\sqrt{y} = \frac{5-Z}{x}$, streamline equation

3.5.2:2 $u = -\frac{y}{b^2}$ $\qquad v = \frac{x}{a^2}$

a.) for an incompressible flow $\bar{\nabla} \cdot \bar{q} = 0$

or $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

since $u = f(y)$ $\qquad \frac{\partial u}{\partial x} = 0$

$v = g(x)$

$\frac{\partial v}{\partial y} = 0$

b.) equations for streamline

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{-dx}{y} b^2 = \frac{dy}{x} a^2$$

$$b^2 \int - x dx = \int a^2 y dy$$

$$\frac{-b^2 x^2}{2} = \frac{a^2 y^2}{2} + C$$

$$C_1 = x^2 b^2 + y^2 a^2$$

$$\left(\frac{C_1}{a^2 b^2}\right) = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$C_2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} \quad \text{where } C_2 = \frac{C_1}{a^2 b^2}$$

Thus the streamlines are a family of ellipses. The special case shown in the problem is for $C_1 = a^2 b^2$ or $C_2 = 1$.

3.6.1:1 Losses are: 3 hp. 6 ft./sec.

6 hp. 9 ft./sec.

losses ≃ [energy loss] x flow

$Kh_1(6) = 3$

$Kh_2(9) = 6$

$\frac{h_1}{h_2}(\frac{6}{9}) = \frac{1}{2}$

$h_2 = \frac{12}{9}(h_1)$; $h_2 = \frac{4}{3} h_1$

$Kh_1 = 3$ $h_2 = 2h_1$ $1 = V^n$

$Kh_2 = 6$ $V_2 = 1.5V_1$

losses ≃ V^n

$L_1 = K(6)^n$

$L_2 = K(9)^n$

$3 = K6^n$

$6 = K9^n$

$2 = (\frac{9}{6})^n$

$1.5^n = 2$

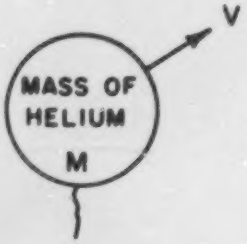
$n > 1$ turbulent

RESOURCE MATERIAL

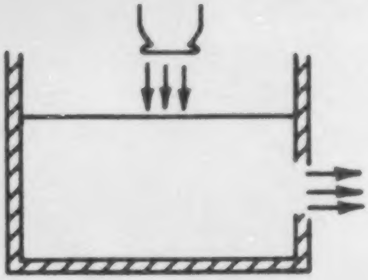
Ref. No.	Chapter	Pages	Other Description
1	3	107-123	Articles 3.1-3.4
4			FM-47 Film Loop
5			FM-48 Film Loop

QUIZ

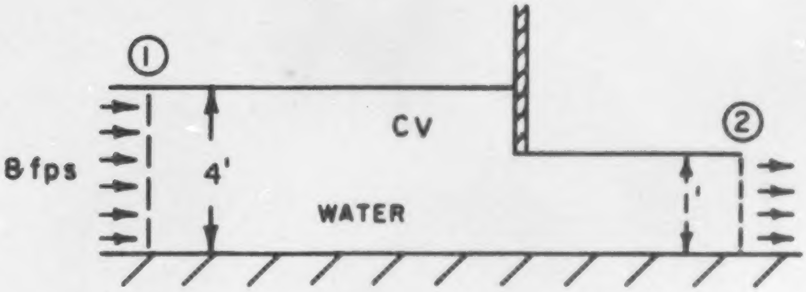
1.) Given the system shown - a balloon filled with helium gas. The conservation of mass equation for this case may be written



2.) The tank shown in the diagram is being filled with liquid. The inflow through the top exceeds the outflow through the side nozzle. For the conditions shown in the diagram, draw a suitable control volume and write the non steady continuity equation. Identify all terms.



3.) For the sluice gate shown, start with the continuity equation in integral form and reduce it to a form applicable to the control volume shown. From this, determine the velocity at section 2. Note: Gate is 8 ft. wide



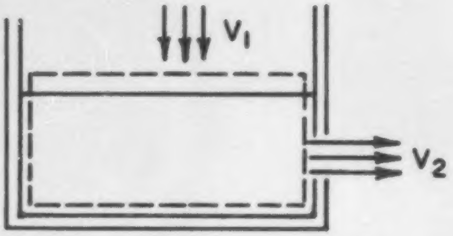
QUIZ ANSWERS

1.) conservation of mass equation is:

$m = C$

$\frac{dm}{dt} = 0$

2.) control volume



non steady continuity equation

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho dv + \int_{CS} \rho(\bar{v} \cdot d\bar{A})$$

identification of terms:

$\frac{\partial}{\partial t}\int_{CV} \rho dv$ is change in CV

$\int_{CS} \rho(\bar{v} \cdot d\bar{A})$ is flow in and flow out over control surface

3.) $$0 = \frac{\partial}{\partial t}\int_{CV} \rho dv + \int_{CS} \rho(\bar{v} \cdot d\bar{A})$$

expanding: $\rho v A_1 = \rho v A_2$

$$v_1 A_1 = v_2 A_2$$

$$8(4)(8) = v_2(1)(8)$$

$$v_2 = 32 \text{ fps}$$

MODULE 4

THE ENERGY EQUATION

SUBJECT MATTER

The second of the three fundamental equations of fluid dynamics is the energy equation which has as its basis the concept of conservation of energy. Intuitively you are aware that in any process, all the energy must be accounted for - which, expressed mathematically, gives us the second of three equations of fluid dynamics.

There are two different forms for the energy equation which will be presented in this module. The first form comes from writing Newton's second law for a small control volume. In order to demonstrate the approach without making the analysis overly complex, we will assume an ideal fluid. Since ideal fluids are inviscid, no shear stresses can exist; therefore, the only forces that we need to consider are gravity forces and pressure forces. Integrating the equation along a streamline gives us an energy equation (recall work = force x distance). Since only mechanical terms are involved, the resulting equation is sometimes referred to as the Mechanical Energy Equation or Bernoulli Equation.

The derivation of the energy equation can also be made from the first law of thermodynamics. The resulting equation, called the general energy equation, is more universal in its scope since losses or irreversibilities are necessarily included. By restricting the analysis to an ideal fluid, the mechanical energy equation results.

While reading the resource material for this module, keep the physical problem in your mind and don't get lost in the mathematics. Note carefully the assumptions made and the resulting form of the equations. You will find the energy equation to be a most powerful tool in the solution of fluid flow problems.

INCREMENTAL OBJECTIVES

At the conclusion of this module you should be able to:

4.1) Define both verbally and mathematically the terms heat, work, potential energy, kinetic energy, and internal energy.

4.2) From recall, verbally and mathematically state the Bernoulli Equation for a streamline together with all the assumptions made in its derivation.

4.3) From recall, state the Bernoulli Equation for a finite section (or collection of streamlines) and solve a given problem with the application of this equation.

4.4) Given the general energy equation, equation 3.7.1 in Reference #1, identify all terms and solve a given problem with the application of this equation.

4.5) Reduce the general energy equation, discussed in objective #4.3, to a form which incorporates all irreversibilities and energy dissipation into a loss term and solve a given problem with the application of this equation.

PROBLEM INDEX

Problem Number	Problem Description
4.1.1:3	Potential Energy
4.1.2:3	Kinetic Energy
4.1.3:3	Mean velocity and kinetic energy
4.2.1:1	Streamline and Bernoulli equation
4.3.1:2	Bernoulli equation
4.3.2:2	Bernoulli equation
4.4.1:1	General energy equation (application)
4.5.1:5	General energy equation (application)
4.5.2:5	General energy equation (application)
4.5.3:5	General energy equation (application)
4.5.4:5	General energy equation (application)
4.5.5:5	General energy equation (application)

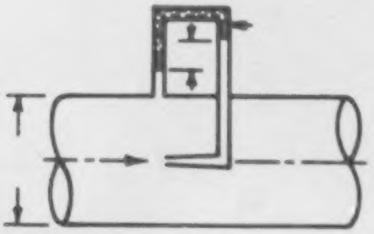
PROBLEM SET #4

4.1.1:3 A standpipe 20 ft. in diameter and 50 ft. high is filled with water. How much potential energy is in this water if the elevation datum is taken 10 ft. below the base of the standpipe?

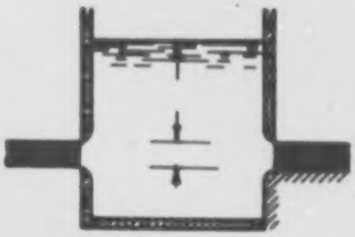
4.1.2:3 What is the kinetic energy in foot-pounds per second of 200 gpm of oil, sp. gr. 0.80, discharging through a 2.0" diameter nozzle?

4.1.3:3 The velocity distribution between two parallel plates separated by a distance a is $u = -10\frac{y}{a} + 20\frac{y}{a}(1 - \frac{y}{a})$ in which u is the velocity component parallel to the plate and y is measured from, and normal to, the lower plate; determine the volume rate of flow and the average velocity. What is the time rate of flow of the kinetic energy between the plates? In what direction is the kinetic energy flowing?

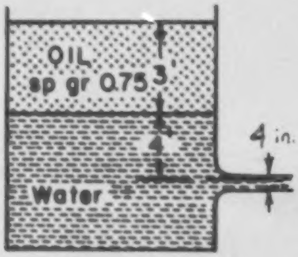
4.2.1:1 In the figure below, determine V for R = 12 inches.



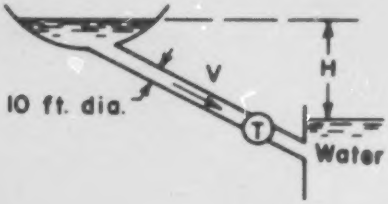
4.3.1:2 In the adjacent figure oil discharges from a "two-dimensional" slot as indicated at A into the air. At B oil discharges from under a gate onto a floor. Neglecting all losses, determine the discharges of A and at B per foot of width. Why do they differ?



4.3.2:2 Neglecting losses, determine the discharge in the figure below.

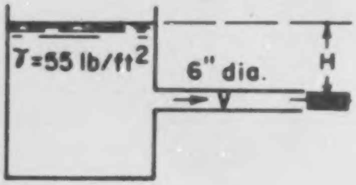

OIL
sp gr 0.75
3'
4'
4 in.
Water

4.4.1:1 Losses through the system shown below are 4 $V^2/2g$. exclusive of the turbine. The turbine is 90 per cent efficient and runs at 240 rpm. To produce 1000 hp. for H = 300 ft., determine the discharge and torque in the turbine shaft. Draw the energy and hydraulic grade lines.

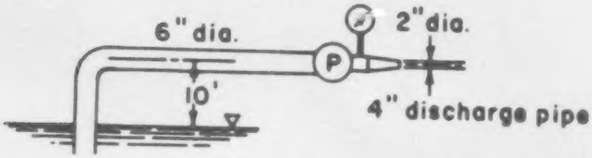

V
H
10 ft. dia.
T
Water

4.5.1:5 The losses in the adjacent figure for $H = 20$ ft. are $\frac{3V^2}{2g}\ \frac{\text{ft-lb}}{\text{lb}}$.

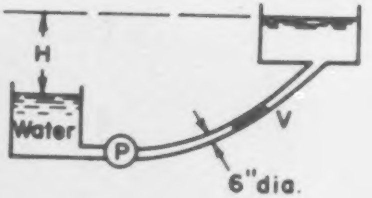
What is the discharge?


γ=55 lb/ft²
6" dia.
V
H

4.5.2:5 The pumping system shown in the figure below must have pressure of 5 psi in the discharge line when cavitation is incipient at the pump inlet. Calculate the length of pipe from the reservoir to the pump for this operating condition if the loss in this pipe can be expressed as $V_1^2/2g$ (0.03) L/D. What horsepower is being supplied to the fluid by the pump? What percentage of this power is being used to overcome losses?

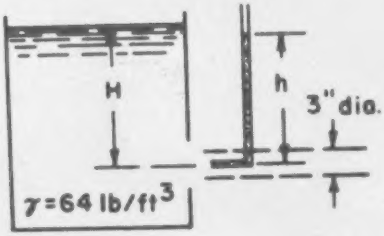


4.5.3:5

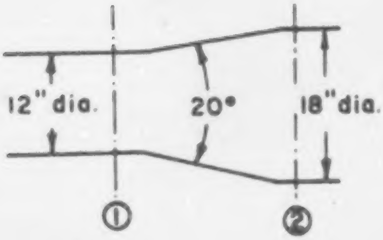


Determine the shaft horsepower for an 80-per-cent-efficient pump to discharge one cfs through the system above. The system losses, exclusive of pump losses, are 10 $V^2/2g$ and H = 50 ft.

4.5.4:5 In the figure below H = 20.0 ft. and h = 19.6 ft. Calculate the discharge and the losses in foot-pounds per pound and in horsepower.


H
h
3" dia.
$\gamma = 64$ lb/ft^3

4.5.5:5 In the figure below, 4 cfs water flows from section 1 to section 2 with losses of $0.4\ (V_1 - V_2)^2/2g$; $p_1 = 10$ psi. Compute p_2, and plot the energy and hydraulic grade lines through the diffuser.

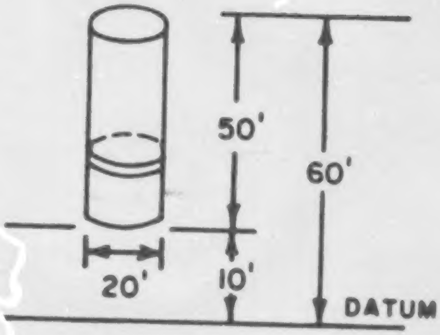

12" dia.
20°
18" dia.
①
②

PROBLEM SET #4--ANSWERS

4.1.1:3 $PE = \Sigma\ \text{wt.}\ (h)$

$$= \int A\,dy\ (\gamma)\ h$$

$$= \int_{10}^{60} \gamma \Pi\ (10)^2\ y\ dy$$



$$= 100\ \Pi\gamma\ \frac{y^2}{2}\ \Big]_{10}^{60}$$

$$= 100\ \Pi\gamma\ \frac{1}{2}\ [60^2 - 10^2]$$

$$= 100\ \Pi\ \frac{\gamma}{2}\ [60 - 10][60 + 10]$$

$$= 100\ \Pi\gamma\ (50)\left(\frac{60 + 10}{2}\right) = \text{wt. x avg. height}$$

$$PE = 100\ (\Pi)(62.4)\ 50\ (35) = 3.43 \times 10^7\ \text{ft-lb.}$$

4.1.2:3 200 gpm oil sp. gr. = .80

Note: gal. x .1337 = ft^3.

or 7.48 gal/ft^3.

2" nozzle

Area = Πr^2

gal/min → cfs $\quad 2.228 \times 10^{-3}$

$200 \times (2.228 \times 10^{-3}) = .4456$ cfs

$$144 \times \frac{.4456}{\Pi}\ \frac{ft^3}{sec} = 20.42\ \text{fps} = \text{velocity}$$

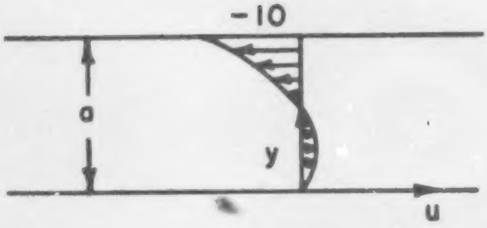
$$K.E. = \frac{1}{2}\ mv^2$$

$$\frac{\text{lb-sec}^2}{\text{ft-sec}} \times \frac{ft^2}{sec^2} \rightarrow \frac{\text{ft-lb}}{sec}$$

$$= \frac{1}{2}\ (.4456)(.8)(1.94) \times (20.42)^2$$

$$= 144.2\ \frac{\text{ft-lb}}{sec}$$

4.1.3:3 $u = -10\frac{y}{a} + 20\frac{y}{a}\left(1 - \frac{y}{a}\right)$



(see plot)

a.) volumetric rate of flow

$$u = -10\frac{y}{a} + 20\frac{y}{a} - 20\frac{y^2}{a^2}$$

$$= 10\frac{y}{a} - 20\frac{y^2}{a^2}$$

$$Q = \int u\,dA = \int_0^a u(1)\,dy$$

Assume 1' depth

y	u
0	0
$\frac{a}{4}$	1.25
$\frac{a}{2}$	0
$\frac{3}{4}$ a	-3.75
a	-10

$$Q = \int_0^a \left(10\frac{y}{a} - 20\frac{y^2}{a^2}\right) dy = \left.\frac{10y^2}{2a} - \frac{20y^3}{3a^2}\right)_0^a$$

$$Q = \frac{5a^2}{a} - \frac{20}{3}a = 5a - 6.67a = -1.67a \text{ (to the left)}$$

b.) Mean Velocity

$$V_{mean} = \left|\frac{Q}{A}\right| = \left|-\frac{1.67a}{a(1)}\right| = 1.67 \text{ fps}$$

c.) Time rate of flow of kinetic energy

$$KE/Time = \int \frac{1}{2}u^2 dm = \frac{1}{2}\int u^2\,(\rho u dA)$$

$$= \frac{1}{2}\rho \int_0^a u^3\,(1)\,dy = \frac{\rho}{2}\int_0^a \left[10\frac{y}{a} - 20\frac{y^2}{a^2}\right]^3 dy$$

$$= \frac{1}{2}\rho \int_0^a \left[1000\frac{y^3}{a^3} - 6000\frac{y^4}{a^4} + 12{,}000\frac{y^5}{a^5} - 8000\frac{y^6}{a^6}\right] dy$$

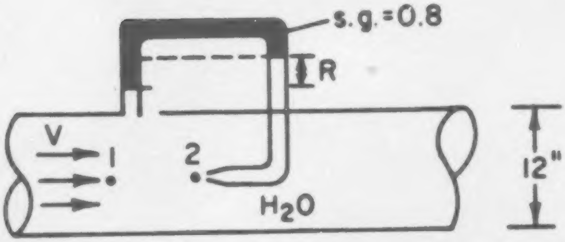
$$= \frac{\rho}{2}\left[\frac{1000}{4}\frac{y^4}{a^3} - \frac{6000}{5}\frac{y^5}{a^4} + \frac{12,000}{6}\frac{y^6}{a^5} - \frac{8000}{7}\frac{y^7}{a^6}\right]_0^a$$

$$= \frac{\rho}{2}\,[250a - 1200a + 2000a - 1143a]$$

$$= -\,93\frac{\rho}{2} = -\,46.5\,\rho$$

d.) Kinetic energy is flowing to the left. This is obvious from the velocity profile shown in the sketch.

4.2.1:1 sp. gr. = 0.8 R = 12"



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 \qquad \frac{V_2^2}{2g} = 0$$

$$P_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma}$$

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\gamma_w} \qquad V_1 = \sqrt{2g\frac{(P_2-P_1)}{\gamma_w}}$$

$$P_c = P_1 - m\gamma_w - R\gamma_m = P_2 - (R + m)\,\gamma_w$$

$$P_1 - R\gamma_m = P_2 - R\gamma_w$$

$$P_2 - P_1 = R\,(\gamma_w - \gamma_m)$$

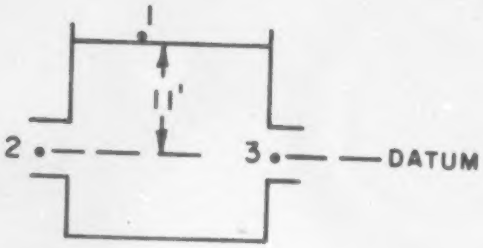
$$\frac{\Delta P}{\gamma_w} = R(1 - \text{sp. gr.})$$

$$V_1 = \sqrt{2gR(1 - \text{sp.gr.})}$$

$$= \sqrt{64.4(1)(1 - .8)} = 3.59 \text{ fps}$$

4.3.1:2 At A $\gamma_1 = \gamma_2 = \gamma_3$

$$\frac{P_1}{\gamma_1} + \frac{V_1^2}{2g} + 1 = \frac{P_2}{\gamma_2} + \frac{V_2^2}{2g} + Z_2$$



$$11 + 0 = \frac{V_2^2}{2g} + 0$$

$$V_2 = \sqrt{22g} = 26.6$$

$$Q = 53.2 \text{ cfs}$$

$$\frac{P_1}{\gamma_1} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma_3} + \frac{V_3^2}{2g} + Z_3$$

$$\frac{P}{\gamma} + 0 + 11 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

$$\frac{P}{\gamma} = 0 \quad , \quad Z_3 = 0$$

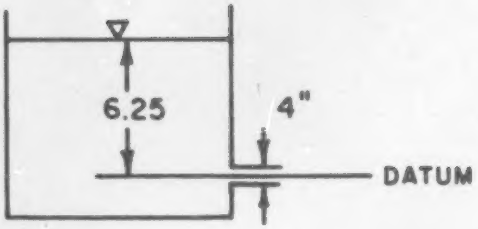
$$11 = (1)\frac{\gamma}{\gamma} + \frac{V_3^2}{2g}$$

$$V_3^2 = 20g$$

$$V_3 = \sqrt{20g}$$

$$Q = 50.75$$

They differ because of the energy distribution. In one case, all energy is kinetic. In the other case, some is still in the form of pressure.



4.3.2:2 equivalent height of water

$h_w \gamma_w = 3\ \gamma_{oil}$

$h_w \gamma_w = 3(\frac{3}{4})\ \gamma_w$

$h = \frac{9}{4} = 2.25$ ft.

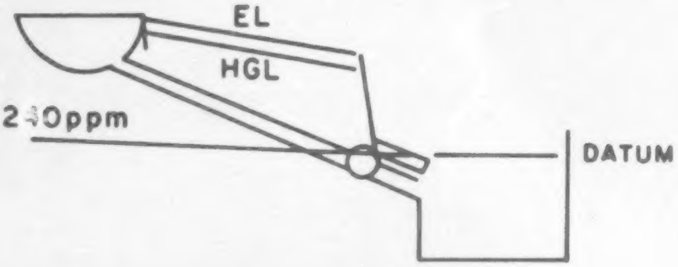
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$6.25 = \frac{V_2^2}{2g} \qquad Q = \frac{\Pi(2)^2}{144} V_2$$

$$V_2 = \sqrt{6.25(2)(32.2)} \qquad = \frac{\Pi(4)}{144}(20.06)$$

$$= 20.06 \text{ fps} \qquad Q = 1.75 \text{ cfs}$$

4.4.1:1 losses $= 4\ \frac{V^2}{2g}$



turbine 90% efficiency @ 240ppm

1000 hp. for it = 300'

1000 hp. = power output

input to turbine $= \frac{1000}{.9} = 1111$ hp. $\qquad P = W_T\ Q\ \frac{\gamma}{550}$

energy equation

$[300 \text{ ft.} + 0 + 0 = W_T + L_{1-2}]$

a.) $300 = W_T + 4\frac{V^2}{2g}$ $\qquad$ b.) $1111 = W_T V\ \frac{A\gamma}{550}$

2nd root

$$300 = \frac{1111(550)}{A\gamma V} + 4\frac{V^2}{2g} \qquad 300 = \frac{124.68}{V} + .0621\ V^2$$

$$300 = \frac{(1111)(550)}{25\Pi(62.4)V} + \frac{4}{2g}\ V^2 \qquad V = \frac{.4156}{300} \text{ fps}$$

$$300 = \frac{124.68}{V} + .0621\ V^2$$

$$300V = 124.68 \quad .0621\ V^3$$

solve for V:

$$V = 69.3\ \text{ft/sec.}$$

$$Q = 25\Pi(69.3) = 5442.8\ \text{cfs}$$

$$\frac{TN}{63{,}000} = HP \quad ; \quad T = \frac{63{,}000(1000)}{240}$$

$$T = 2.625 \times 10^5\ \text{in-lb.}$$

$$Q = 25\Pi(.4156)$$

$$= 32.6\ \text{ft}^3/\text{sec.}$$

$$\frac{TN}{63{,}000} = 1000$$

$$T = \frac{1000(63{,}000)}{240}$$

$$\text{Torque} = 21{,}875\ \text{ft-lb.}$$

Which solution actually exists?

a.) $W_t = \frac{124.68}{69.3} = 1.799\ \frac{\text{ft-lb}}{\text{lb}}$

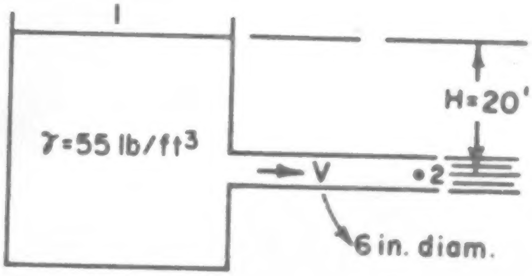
$$\text{losses} = \frac{2(69.3)^2}{32.2} = 298\ \frac{\text{ft-lb}}{\text{lb}}$$

b.) $W_t = \frac{124.68}{.4156} = 300\ \frac{\text{ft-lb}}{\text{lb}}$

$$\text{losses} = \frac{4(.4156)^2}{64.4} = .0107\ \frac{\text{ft-lb}}{\text{lb}}$$

prefer the former

4.5.1:5



$$\text{losses} = 3\frac{V^2}{2g}$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + L_{1-2}$$

$$0 + 0 + 20 =) + \alpha\frac{V_2^2}{2g} + 0 + 3\frac{V^2}{2g}$$

assume $\alpha = 1$ $\qquad 20 = \frac{V_2^2}{2g}(4)$

$$V_2^2 = 5(2g) = 10g$$

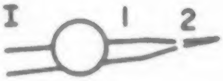
$$V_2 = \sqrt{10g}$$

$$V_2 = \sqrt{322} = 17.94$$

$Q = .196\ (17.94) = 3.52\ \text{ft}^3/\text{sec.}$

4.5.2:5 5 psig

@ 70° $\qquad P_v = (.84)(62.4)$ psfa



a.)

$$\frac{P_1}{\gamma_1} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_2} + \frac{V_2^2}{2g} + Z_2$$

$$\gamma_1 = \gamma_2$$

$$\frac{5(144)}{62.4} + \frac{V_1^2}{2g} + 0 = 0 + \frac{V_2^2}{2g} + 0$$

$$\frac{5(144)}{62.4} = 16\frac{V_1^2}{2g} - \frac{V_1^2}{2g} = 15\frac{V_1^2}{2g}$$

$$V_1 = \sqrt{\frac{10g(144)}{15(62.4)}} = \sqrt{\frac{144(64.4)}{3(62.4)}}$$

$V_1 = 7.04$ fps

$$V_1A_1 = V_2A_2$$

$$V_1(16) = V_2(4)$$

$$V_1 4 = V_2$$

$$V_2^2 = 16\ V_1^2$$

Pump cannot change velocity $\therefore$ $V_1(36) = 7.04\ (16)$

$$V_1 = \frac{16}{36}(7.04) = 3.13 \text{ fps}$$

energy equation from surface to I:

$$\frac{P_s}{\gamma_s} + \frac{V_s^2}{2g} + Z_s = \frac{P_I}{\gamma} + \frac{V_I^2}{2g} + Z_1 + L_{s \to I}$$

$$\frac{P_{BAR}}{\gamma_w} + 0 + 0 = (.84) + \frac{9.79}{64.4} + 10 + \frac{9.79}{64.4}(.03)(2)(L)$$

assume 14.7 psia for barometric pressure

$$\frac{14.7(144)}{62.4} = (.84) + 10 + (.1521) + .152\,(.06)\,L$$

$$(.79)(.06)\,L = 33.9 - 10.992 = 22.9 \quad ; \; L = \frac{22.9}{(.152)(.06)} = 2511 \text{ ft.}$$

b.) HP:

$$-W_p = [E_1 - E_I] = [\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1] - \frac{P_I}{\gamma} - \frac{V_I^2}{2g} - Z_I$$

$$= [\frac{19.7(144)}{62.4} - .84] + \frac{7.04^2 - 3.13^2}{2g} = 45.24$$

$$W_p = 45.24 \text{ ft-lb/lb.}$$

$$\text{Power} = \frac{45.24 \times (\frac{\Pi}{16})(3.13)(62.4)}{550} = 3.15$$

4.5.3:5 80% efficient pump, $Q = 1$ cfs, $H = 50$ ft., losses $= 10\,\frac{V^2}{2g}$

energy equation

$$-\text{ shaft work} = (E_2 - E_1)$$

$$E_1 = E_2 + W_s \to \frac{\text{energy}}{\text{mass}} \qquad \text{or}$$

$$P_1 = P_2 + W_s$$

$$0 = W_s + 0 + 0 + H + L_{1-2}$$

work done on is negative;

work done by is positive.

$$W_s = -(H + L_{1-2}) = -50 - 10\frac{V^2}{2g}$$

continuity equation

$Q = 1 = VA \qquad V = \frac{1}{A} = \frac{1}{.196} = 5.10$ fps

$W_s = -50 - \frac{10(5.10)^2}{64.4} = 54.04 \frac{\text{ft-lb}}{\text{lb}}$

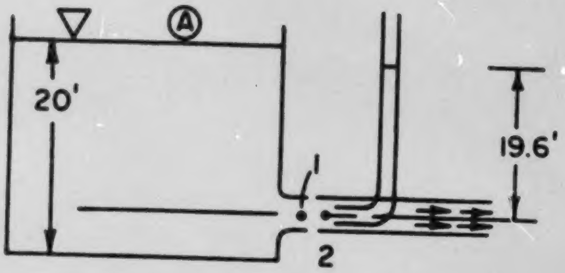
$\text{Power} = \frac{(54.04) \times (62.4)}{550} \times 1 \qquad \frac{\text{ft-lb}}{\text{lb}} \frac{\text{lb}}{\text{sec}}$

Ideal = 6.13 hp.

$\text{Actual} = \frac{6.13}{.80} = 7.66$ hp.

4.5.4:5 write energy equation

from 1 to 2



$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{V_1^2}{2g} = \frac{P_2}{\gamma} = \frac{19.6(64)}{64}$$

$V_1 = \sqrt{19.6(64.4)} = 35.5$ fps

$\frac{P_1}{\gamma} = 0$

$Z_1 = 0$

$Z_2 = 0$

$\frac{V_2^2}{2g} = 0$

Discharge $Q = VA = 35.5 \times \text{Area} = 35.5 \times (.0491) = 1.74$ cfs losses

write energy equation from A to 2

$$\frac{P_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + L_{A-2}$$

$$\frac{P_A}{\gamma} = 0$$

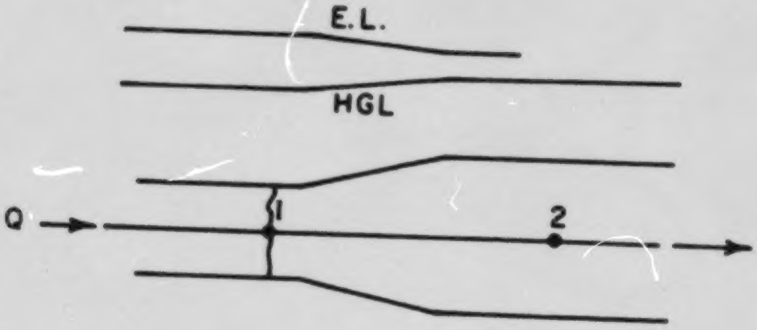
$$0 + 20 + 0 = 19.6 + 0 + 0 + L_{A-2}$$

$$\text{Losses}_{A-2} = .4 \; \frac{\text{ft-lb}}{\text{lb}}$$

$$\text{Horsepower} = \frac{L_{A-1} \cdot \gamma Q}{550} = \frac{0.4(64)(1.74)}{550}$$

HP = .081

4.5.5:5 Q = 4 cfs



$$L_{1-2} = .4\frac{(V_1-V_2)}{2g}$$

P_1 - 10 psi

P_2 = ?

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + L_{1-2}$$

12" D, A = .785

18" D, A = 1.767

$$V_1 = \frac{4}{.785} = 5.09 \qquad \frac{V_1^2}{64.4} = .403 \; \frac{\text{ft-lb}}{\text{lb}}$$

$$V_2 = \frac{4}{1.767} = 2.26 \qquad \frac{V_2^2}{64.4} = .08$$

$$\frac{10(144)}{62.4} + .403 = \frac{P_2}{62.4} + (.08) + \frac{(.4)(5.09 - 2.26)^2}{64.4}$$

$$R_e = \frac{VD}{N_u}$$

$$= \frac{5.09(1)}{1} 10^5$$

$$\frac{10(144)}{62.4} + .403 - .08 - .049 = \frac{P_2(144)}{62.4}$$

$$P_2 = 23.35 \frac{(62.4)}{144} = 10.12 \text{ psig}$$

$E_1 = 23.47 \frac{\text{ft-lb}}{\text{lb}}$ $\quad$ $(\frac{P_1}{\gamma} + z)_1 = 23.07 \frac{\text{ft-lb}}{\text{lb}}$

$E_2 = 23.43 \frac{\text{ft-lb}}{\text{lb}}$ $\quad$ $(\frac{P}{\gamma} + z)_2 = 23.35 \frac{\text{ft-lb}}{\text{lb}}$

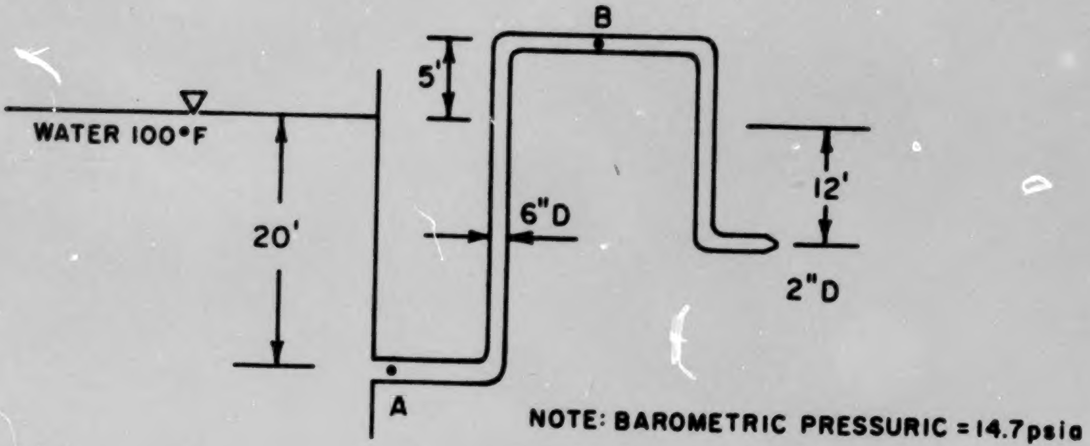
RESOURCE MATERIAL

Ref. No.	Chapter	Pages	Other Description
1	3	123-139	Articles 3.5-3.10
6			FM-36 Film Loop

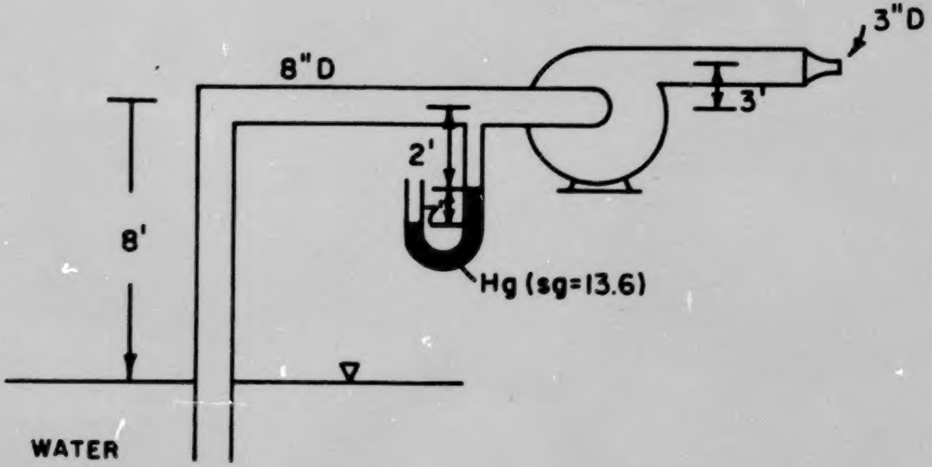
QUIZ

1. a) Calculate the rate of flow (cfs) through this pipeline (neglecting losses) and the pressure (in lb/in^2) at points A and B.

   b) How much higher could point B be raised and still have the fluid flow?


B
5'
WATER 100°F
20'
6"D
12'
2"D
A
NOTE: BAROMETRIC PRESSURIC = 14.7psia

2. Calculate the pump horsepower for the system shown below.


3"D
8"D
3'
2'
8'
Hg (sg=13.6)
WATER

QUIZ ANSWERS

1.) Energy equation from 1 to 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$0 + 0 + 20 = 0 + \frac{V_2^2}{2g} + 8$$

$$12 = \frac{V_2^2}{2g} \qquad V_2 = \sqrt{24g} = 27.8 \text{ fps}$$

$$Q = (.0218)(27.8) = 0.606 \text{ cfs}$$

Pressure at A

Energy equation from 1 to A

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_A}{\gamma} + \frac{V_6^2}{2g} + Z$$

$$\frac{P_1}{\gamma} = 0 \qquad Z = 0$$

$$\frac{V_1^2}{2g} = 0$$

$$V_{6''}\,(.196) = V_{2''}\,(.0218)$$

$$V_6 = 27.8\,\frac{(.0218)}{.196}$$

$$V_6 = 3.092 \text{ fps}$$

$$\frac{V_6^2}{2g} = .148$$

$$\therefore \quad 20 = \frac{P_A}{\gamma} + .148$$

$$19.85 = \frac{P_A}{\gamma} \quad ; \quad P_A = \frac{62(19.85)}{144} = 8.55 \text{ psig}$$

Pressure at B

Energy equation from 1 to B

$$20 = 25 + .148 + \frac{P_B}{\gamma}$$

$$-5.148 = \frac{P_B}{\gamma}$$

$$P_B = -\frac{5.148(62)}{144} = -2.22 \text{ psig}$$

could raise B until $\frac{P_B}{\gamma}$ -31.81

$20 = Z + 148 - 31.81$

$51.66 = Z$ or could be raised another 46.66'

2.) At pump inlet

$$P = -\frac{7}{12}(13.6)(62.4) - 2(62.4)$$

$$\frac{P}{\gamma} = -\frac{7}{12}(13.6) - 2 = -9.93 \text{ ft.}$$

Energy equation from 1 to A

$$\frac{P}{\gamma} + Z + \frac{V^2}{2g}$$

$$0 + 0 + 0 = -9.93 + 8 + \frac{V_8^{\ 2}}{2g}$$

$$\frac{V_8^{\ 2}}{2g} = 1.93$$

$$V_8 = \sqrt{1.93(2g)} = \sqrt{124.29} = 11.15 \text{ fps}$$

Energy equation from A to 2

$$\frac{P_A}{\gamma} + \frac{V_A^{\ 2}}{2g} + Z_A + W_p = \frac{P_2}{\gamma} + \frac{V_2^{\ 2}}{2g} + Z_2$$

$$-9.93 + 1.93 + 8 + W_p = 0 + \frac{V_2^{\ 2}}{2g} + 3$$

Continuity equation

$$V_3A_3 = V_8A_8$$

$$.0491\ V_3 = V_8\ (349)$$

$$V_3 = 11.15\ \frac{(.349)}{.0491} = 79.25\ \text{fps}$$

$$\frac{V_3^{\ 2}}{2g} = 97.53$$

$$W_p = 97.53 = \# = 100.53\ \frac{\text{ft-lb}}{\text{lb}}$$

$$HP = \frac{W_pQ\gamma}{550} = \frac{(100.53)(11.15)(.349)(62.4)}{550}$$

$$= 44.38\ \text{horsepower}$$

MODULE 5

THE MOMENTUM EQUATION

SUBJECT MATTER

The third of the fundamental equations of fluid dynamics is the momentum equation. It is based on Newton's law of motion, which you will recall, may be written as

$$\Sigma\bar{F} = m\bar{a}$$

$$= \frac{d}{dt}(m\bar{v})$$

where $\Sigma\bar{F}$ represents the various forces acting on the system, m is the mass of the system, and $\bar{a}$ the acceleration.. Note for a particle or rigid body the first form may be more useful whereas for fluids the second form is most convenient. Emphasis is placed on the term system here because Newton's law is written for a Lagrangian reference frame.

A comparable form of Newton's equation may be obtained for a control volume by using the equation which you have previously derived which relates the time rate of change of momentum for a system to that for a control volume. In equation form

$$\Sigma\bar{F} = \frac{\partial}{\partial t}\int_{cv} \bar{v}\rho dV + \int_{cs} \rho\bar{v}\,(\bar{v} \cdot d\bar{A})$$

Unlike the continuity and energy equations which are scalar equations, the momentum equation is a vector equation and is equivalent to three scalar equations - one for each coordinate direction.

The problem selection for this module will demonstrate the usefulness of the momentum equation whenever forces are to be computed in fluid flow, as for example on turbine blades, pump impellers, propeller blades or jet engines.

INCREMENTAL OBJECTIVES

At the conclusion of this module you should be able to:

5.1) From recall, write the general intergral form of the momentum equation for a system and for a control volume.

5.2) Reduce non-steady flow problems to equivalent steady flow problems for application of the momentum equation by proper choice of control volume.

5.3) Reduce the general momentum equation to a form suitable for computing the thrust on a rocket with constant fuel burning rate.

5.4) Reduce the general momentum equation to a form suitable for steady flow devices such as jet engine, propellers, turbine blades (fixed as well as moving), and open channel structures.

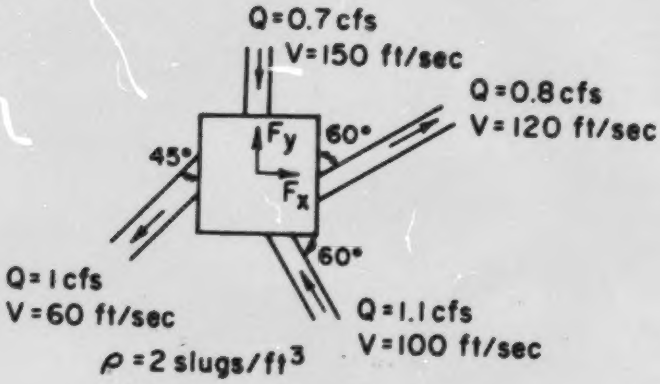
5.5) Given a problem in fluid mechanics, state the usefulness of the momentum equation (conceptually as well as mathematically) in the solution of that problem.

PROBLEM INDEX

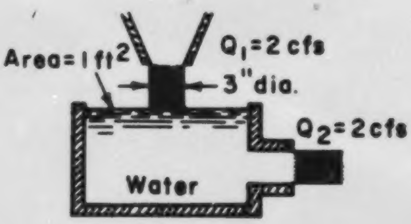
Problem Number	Problem Description
5.4.1:2	Boundary forces for multiple inflow-outflow control volume
5.4.2:12	"Apparent" weight due to flow into a weigh tank
5.4.3:12	Analysis of the forces on a 90° pipe bend due to internal water flow
5.4.4:12	Computation of force on a sluice gate caused by open channel flow
5.4.5:12	Use of the momentum equation to analyze the flow through an airplane propeller
5.4.6:12 (5.2.1:4)	Computation of thrust developed by a jet aircraft
5.4.7:12 (5.2.2:4)	Computation of thrust developed by a jet boat
5.4.8:12	Computation of force on a turbine blade due to a fluid jet
5.4.9:12 (5.2.3:4)	Two dimensional analysis of force on a moving vane due to a fluid jet
5.4.10:12 (5.2.4:4)	Computation of horsepower developed by a turbine blade
5.4.11:12	Hydraulic jump analysis
5.4.12:12	Condition of a hydraulic jump downstream from a spillway

PROBLEM SET #5

5.4.1:12 Given the figure below, determine what force components F_x, F_y are required to hold the "black box" stationary. All pressures are zero gage.

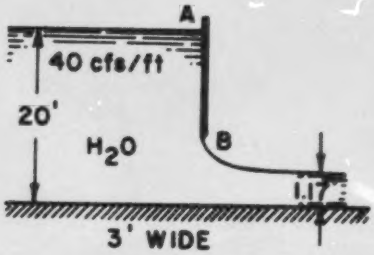

Q=0.7 cfs
V=150 ft/sec
Q=0.8 cfs
V=120 ft/sec
60°
45°
Fy
Fx
60°
Q=1 cfs
V=60 ft/sec
Q=1.1 cfs
V=100 ft/sec
ρ=2 slugs/ft³

5.4.2:12 How much is the apparent weight of the tank full of water (figure below) increased by the steady jet flow into the tank?


Area=1 ft²
Q₁=2 cfs
3" dia.
Q₂=2 cfs
Water

5.4.3:12 20 ft^3/sec. of water flows through an 18" diameter pipeline that contains a horizontal 90° bend. The pressure at the entrance to the bend is 10 psi. Determine the force components, parallel and normal to the approach velocity, required to hold the bend in place. Neglect losses.

5.4.4:12 Determine the net force on the sluice gate shown in the figure below. Neglect losses. By noting that the pressure at A and B is atmospheric, sketch the pressure distribution on the surface AB. Is it a hydrostatic distribution? How is it related to the force just calculated?

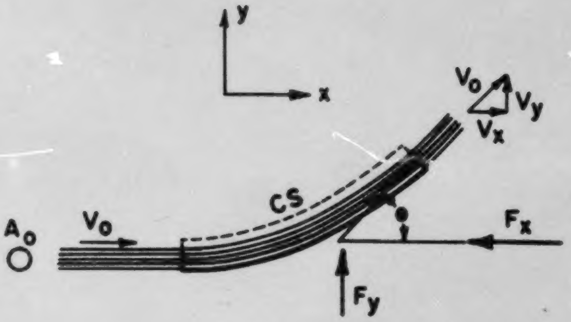


5.4.5:12 An airplane with propeller diameter of 8.0 ft. travels through still air (ρ = 0.0022 slug/ft^3) at 210 mph. The speed of air through the plane of the propeller is 280 mph. relative to the airplane. Calculate (a) the thrust on the plane, (b) the kinetic energy per second remaining in the slipstream, (c) the theoretical horsepower required to drive the propeller, (d) the propeller efficiency, and (c) the pressure difference across the blades.

5.4.6:12 A jet-propelled airplane traveling 600 mph. takes in 87 lb_m/sec. air and discharges it at 1800 ft/sec. relative to the airplane. Neglecting the weight of fuel, what thrust is produced?

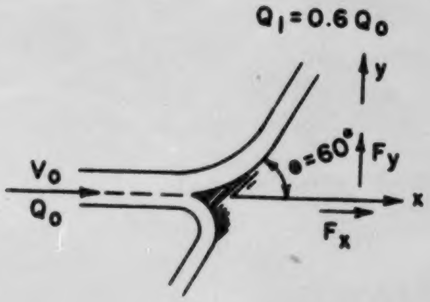
5.4.7:12 Determine the size of jet pipe and the theoretical horsepower necessary to produce a thrust of 2000 lb. on a boat moving 40 ft/sec. when the propulsive efficiency is 68 per cent.

5.4.8:12 Determine the resultant force exerted on the vane in the figure below. $A_o = 0.1\ \text{ft}^2$; $V_o = 100$ ft/sec.; $\theta = 60^\circ$, $\gamma = 55\ \text{lb/ft}^3$. How can the line of action be determined?



5.4.9:12 (5.2.3:4)

If the vane of the figure below moves in the x-direction at u = 40 ft/sec, for Q = 2 cfs, $\rho = 1.935\ \text{slugs/ft}^3$, $V_o = 150$ ft/sec, what are the force components F_x, F_y?

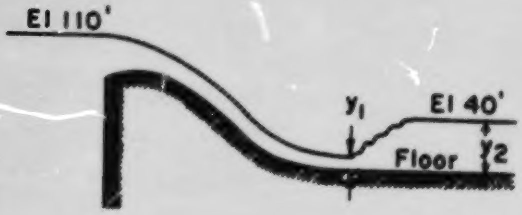


5.4.10:12 (5.2.4:4)

What horsepower can be developed from (a) a single vane and (b) a series of vanes when $A_o = 10\ \text{in.}^2$, $V_o = 240$ ft/sec., u = 80 ft/sec., and $\theta = 173^\circ$, for water flowing?

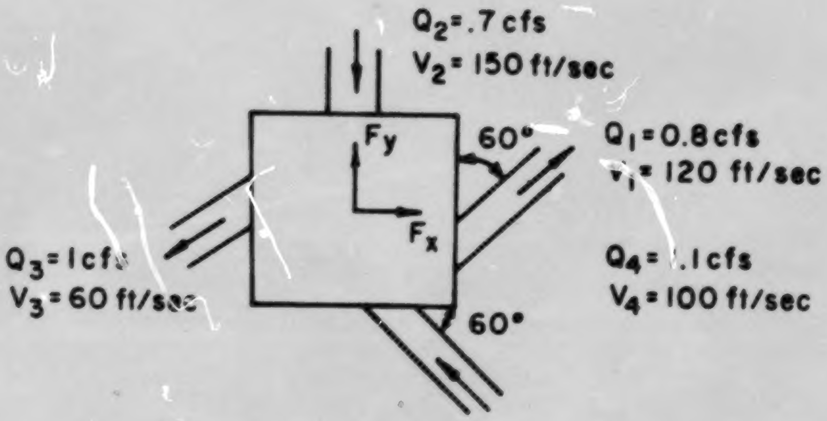
5.4.11:12 A jump occurs in a 20' wide channel carrying 600 cfs water at a depth of 1 ft. Determine y_2, V_2, and the losses in ft.-pounds per pound, in horsepower, and in BTU per pound.

5.4.12:12 For losses down the spillway (figure below) of 8 ft-lb/lb and discharge per foot of 120 cfs, determine the floor elevation for the jump to occur.


El 110'
y_1
El 40'
Floor
2

PROBLEM SET #5--ANSWERS

5.4.1:12


Q_2=.7 cfs
V_2= 150 ft/sec
F_y
F_x
60°
Q_1=0.8cfs
V_1= 120 ft/sec
Q_4= 1.1cfs
V_4=100 ft/sec
Q_3= 1cfs
V_3=60 ft/sec
60°

$$F_x = \rho Q_1 V_1 \sin 60 + \rho Q_3 (-V_3 \cos 45^\circ) - \rho Q_2(0) - \rho Q_4 (-V_4 \cos 60^\circ)$$

$$= \rho[(.8)(120)(.866) - 1(60)(.807) - 0 + 1.1(100)(\tfrac{1}{2})]$$

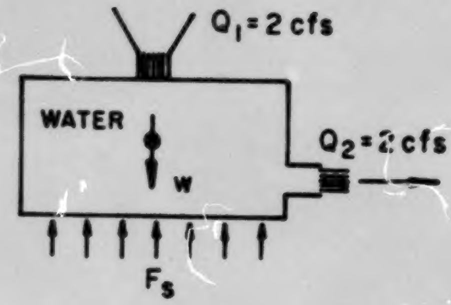
$$= 2(95.72) = 191.43 \text{ lb.}$$

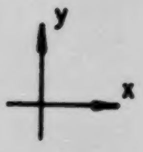
$$F_y = \rho[(.8)(120)\tfrac{1}{2} + 1(-60)(.707) - .7(-150) - 1.1(+100) \sin 60^\circ]$$

$$= 2[48 - 42.4 + 105 - 95.26]$$

$$= 2(15.34) = 30.68 \text{ lb.}$$

5.4.2:12


Q₁ = 2 cfs
WATER
W
Q₂ = 2 cfs
Fₛ



$$\Sigma F_y = \int \rho V_y (\bar{v} \cdot d\bar{A})$$

$$V = \frac{2}{.0491} = 40.7 \text{ ft/sec}$$

$$V = \frac{Q}{A}$$

$$F_s - W = \rho Q_{out} V_{y\,out} - \rho Q_{in} V_{y\,in}$$

$$= -\rho Q_{in}(-V) = +\rho Q_{in} V_{in}$$

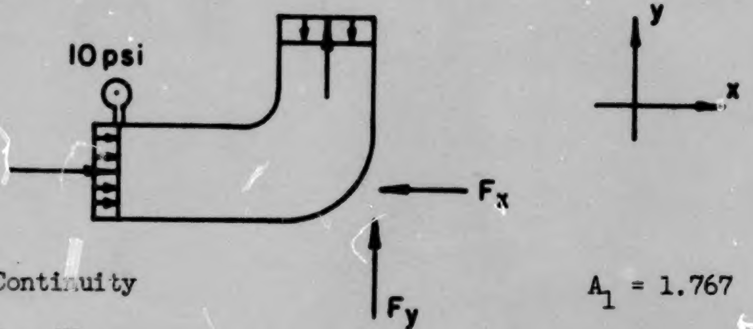
$$F_s - W = 1.94(2)(20.37)$$

$$F_s = W + 2(1.94)(40.73) = 158\#$$

$$= W + 1.94 \frac{4}{(.0491)}$$

5.4.3:12 $Q = 20$ cfs

18" diameter pipeline



1.) Continuity

$A_1 = 1.767$

$Q = AV$

$$V = \frac{Q}{A} = \frac{20}{1.767} = 11.3 \text{ ft/sec.}$$

2.) Energy

$$\frac{P}{\gamma})_1 + \frac{V_1^{\,2}}{2g} = \frac{P}{\gamma})_2 + \frac{V_2^{\,2}}{2g} \quad \therefore \; V_1 = V_2$$

$$P_1 = P_2$$

3.) Momentum

$$10(144)\,1.767 - F_x = \rho Q\,[V_{2x} - V_{1x}]$$

$$1440\,(1.767) - F_x = 1.94(20)\,[0 - 11.3] = -\,1.94(20)(11.3)$$

$$F_x = 1440\,(1.767) + 1.94(20)(11.3)$$

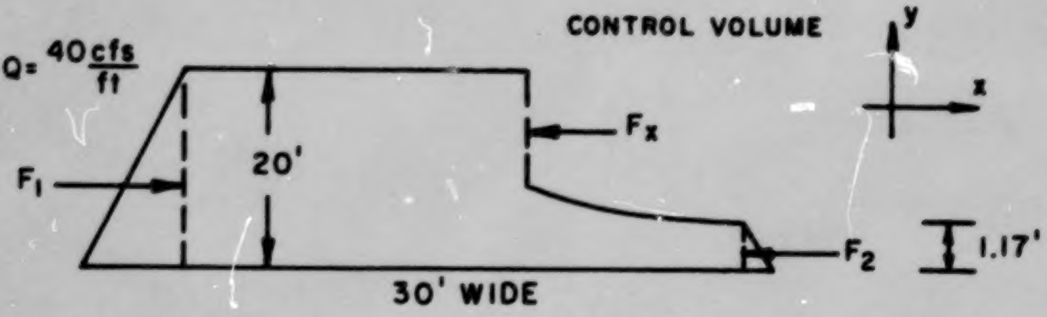
$$= 2983 \text{ lb.}$$

$$-1440\,(1.767) + F_y = \rho Q\,(V_{2y} - 0)$$

$$-2544.5 + F_y = 1.94(20)(11.3) = +438.4$$

$$F_y = +2983 \text{ lb.}$$

5.4.4:12


CONTROL VOLUME
$Q = 40 \frac{cfs}{ft}$
20'
F_1
F_x
F_2
1.17'
y
x
30' WIDE

1.) Continuity equation

$V_1 = \frac{40}{20} = 2$ ft/sec. $\qquad V_2 = \frac{40}{1.17} = 34.19$ ft/sec.

2.) From hydrostatics

$$F_1 = \gamma h_1 A_1 = 62.4(10)(20 \times 30) = 374{,}400 \text{ lb.}$$

$$F_2 = \gamma h_2 A_2 = 62.4\left(\frac{1.17}{2}\right)(30 \times 1.17) = 1281 \text{ lb.}$$

3.) Momentum equation

$$-F_x + F_1 - F_2 = \rho Q\,[V_{2x} - V_{1x}]$$

$$-F_x - 1281 + 374{,}400 = 1.94(40)(30)\,[34.19 - 2]$$

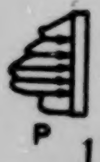
$$F_x = 374{,}400 - 74{,}938 - 1281 = 298{,}181 \text{ lb.}$$

Pressure distribution is not hydrostatic since

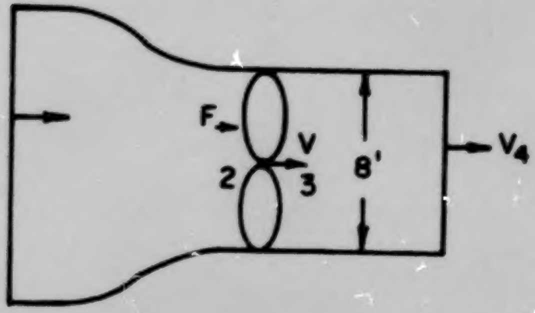
$P_A = p_A = 0$ psig

The integral of the pressure distribution on the plate AB is the force F_x.

$$F_x = \int^{\text{Area}} p\, dA$$



5.4.5:12 still air $\rho = .0022 \frac{\text{slugs}}{\text{ft}^3}$



$V_1 = 210$ mph

$= 308$ fps

$V = 280 \times \frac{44}{30} = 410.67$ fps

$Q = V_8A_8 = (410.67)\,\pi\,(4)^2$ $\qquad F = \rho Q\,(V_4 - V_1)$

$V = \frac{V_1 + V_4}{2}$ $\qquad V_4 = 2V - V_1 = 2(410.67) - 308 = 513$ fps

$F = (.0022)(20{,}643)\,[513 - 308]$

a.) $F = 9325$ lb.

b.) $KE = \frac{1}{2}\rho Q\,(V_4 - V_1)^2 = \frac{1}{2}(.0022)(20{,}643)(513 - 308)^2$

$= 954{,}274$ ft-lb/sec.

c.) useful power = FV

$= \frac{9325\,(308)}{550} = 5222$ hp.

power in exit stream $= \frac{954{,}274}{550} = 1735$ hp

total horsepower $= 6957$ hp

d.) propeller efficiency $= \frac{V_1}{V} = \frac{308}{410.67} = .75$ or 75%

e.) pressure difference across blade

$P_3 - P_2 = \frac{F}{A} = \frac{9325}{16\pi}$

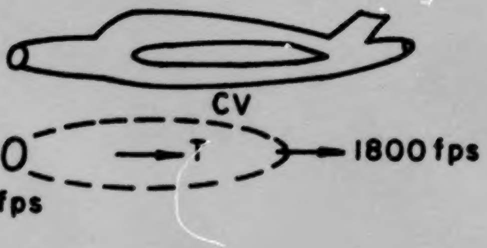
$= 185.5$ lb/ft^2

$= 1.29$ psig

5.4.6:12 Jet plane travels 600 mph

87 lb_m/sec

By superposing a velocity of 600 mph to the right, reduce the plane to rest. See resulting control volume.

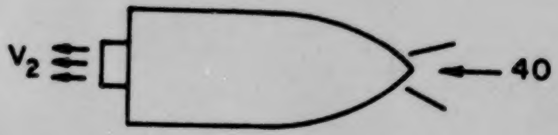


$$T = \rho Q\,(V_2 - V_1) = \frac{.87}{32.2}\,(1800 - 880) = 2485 \text{ lb.}$$

5.4.7:12 (5.2.2:4)

size jet , thrust of 2000 lb

V_B = 40 fps , propulsive efficiency = 68%



$$e = \frac{1}{1 + \frac{(V_2 - V_1)}{2V_1}}$$

$$.68 = \frac{1}{\left(1 + \frac{(V_2 - 40)}{80}\right)}$$

$$\left(.68 + .68\,\frac{(V_2 - 40)}{80}\right) = 1$$ solving, V_2 = 77.65 fps

Thrust of 2000 lb.

$$2000 = (1.94)(V_2A_2)(V_2 - V_1)$$

$$2000 = (1.94)(77.65)(A_2)(37.65)$$

$$A_2 = \frac{2000}{1.94(77.65)(37.65)} = .3562 \text{ ft}^2.$$

$$\frac{\Pi D^2}{4} = .3562$$

$$D = \sqrt{\frac{4}{\Pi}(.3562)} = .673 \text{ ft.}$$

$$D = 8.08 \text{ in.}$$

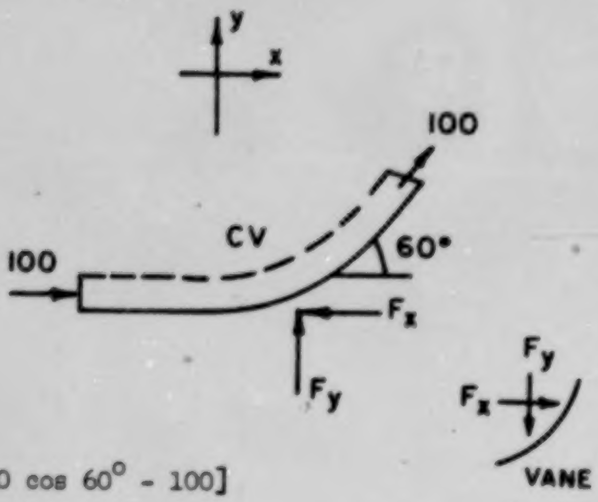
$$\text{Horsepower} = \frac{\text{output}}{\text{eff.}} = \frac{FV}{.68} = \frac{2000(40)}{550(.68)} = 214 \text{ HP}$$

5.4.8:12 $A_o = 0.1 \text{ ft}^2.$

$V_o = 100$ ft/sec.

$\theta = 60^\circ$

$\gamma = 55 \text{ lb/ft}^3.$



$$\Sigma F_x = \rho Q\,(V_{2x} - V_{1x})$$

$$-F_x = \frac{(55)}{32.2}(.1)(100)\,[100 \cos 60^\circ - 100]$$

$$-F_x = \frac{55(10)}{32.2}(-50) = \frac{-550(50)}{32.2} = -854 \quad , \quad F_x = 854 \text{ lb.}$$

$$\Sigma F_y = \rho Q\,(V_{2y} - V_{1y})$$

$$F_y = \left(\frac{55}{32.2}\right)(.1)(100)\,[100 \sin 60^\circ - 0] = \frac{550}{32.2}(86.6) = 1479 \text{ lb.}$$

Resultant Force $\bar{F} = \underset{\rightarrow}{F_x\bar{i}} - \underset{\downarrow}{F_y\bar{j}}$

$$= 854 + 1479$$

$$= 1708 \text{ lb.}$$

Line of action could be determined by using the moment of momentum equation.

5.4.9:12 (5.2.3:4)

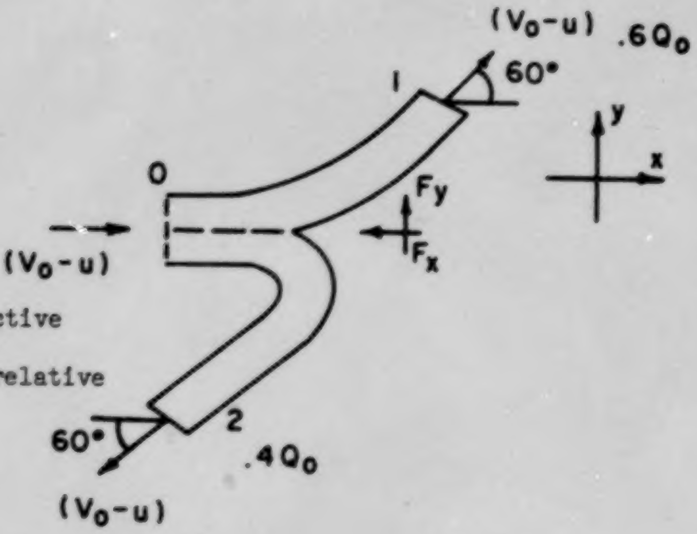
$u = 40$ fps

$Q = 2$ cfs

$V_o = 150$ fps

$\rho = 1.935$ slugs/ft^3.

For a single blade effective Q must be based on the relative velocity.



$Q_o = V_oA_o$

$$A_o = \frac{Q_o}{V_o} = \frac{2}{150} = .0133 \text{ ft}^2$$

$$Q_{rel} = (.0133)(150 - 40) = 110(.0133) = 1.467 \text{ ft}^3/\text{sec.}$$

x) $-F_x = \rho Q_1 V_{1x} + \rho Q_2 V_{2x} - \rho Q_o V_{ox}$

$$-F_x = 1.935(.6Q_o)(V_o - u) \cos 60^\circ + 1.935(.4Q_o)(-)(V_o - u) \cos 60^\circ$$
$$-1.935\, Q_o(V_o - u)$$

$$-F_x = 1.935\,(1.467)(V_o - u)\,[.6 \cos 60^\circ - .4 \cos 60^\circ - 1]$$

$F_x = 218$ lb. (directed as shown as above)

y) $F_y = (\rho Q_1)\, V_{1y} + \rho Q_2 V_{2y} - \rho Q_o V_{oy}$

where $V_{oy} = 0$

$$F_y = \rho\,(.6)(1.467)(V_o - u)\sin 60^\circ - \rho\,(.4)(1.467)(V_o - u)\sin 60^\circ$$

$F_y = 54.1$ lb. (directed as shown)

5.4.10:12 (5.2.4:4)

a.) single vane

$A_o = 10\ \text{in}^2$.

$V_o = 240$ ft/sec.

$u = 80$ ft/sec.

$\theta = 173^\circ$ water flowing

single blade

$$-F_x = \rho Q_{rel}\,[-(V_o-u)\cos\theta - (V_o-u)]$$

$$= 1.94\,(V_o-u)\,\frac{10}{144}\,[(V_o-u)(1 - \cos\theta)]$$

$$= 1.94\,(160)^2\,\frac{10}{144}\,(1 + .9925)$$

$$= 6872 \text{ lb.}$$

$$\text{power} = \frac{6872(80)}{550} = 999.5$$

b.) series of vanes

$$F_x = (1.94)(240)(160)\,\frac{10}{144}\,(1.9925)$$

$$\text{Power} = \frac{1.94(240)(1600)(1.9925)(80)}{144 \qquad 550}$$

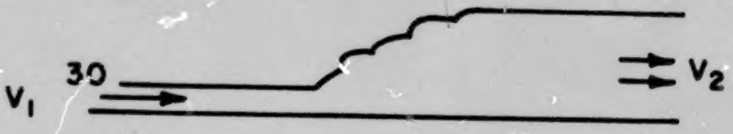
$= 1499$ hp

Note: angle of 173° is unrealistic - should be approximately 150° in practice.

5.4.11:12 20' wide channel

600 cfs

$y_1 = 1$ ft.



$V_1 = \frac{600}{20} = 30$ ft/sec.

$F_1 = \frac{V_1^2}{gy_1} = \frac{900}{(32.2)} \sim 25^+$ super critical

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + 2V_1^2\frac{y_1}{g}} \quad = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(900)}{32.2}}$$

$= 6.99$ ft.

$h_i = \frac{(y_2 - y_1)^3}{4y_1y_2} \qquad \frac{\text{ft-lb}}{\text{lb}}$

$= \frac{(5.99)^3}{4(6.99)(1)} = 7.696 \qquad \frac{\text{ft-lb}}{\text{lb}}$

$\text{Power} = \frac{7.696(62.4)(600)}{550} = 524$ hp.

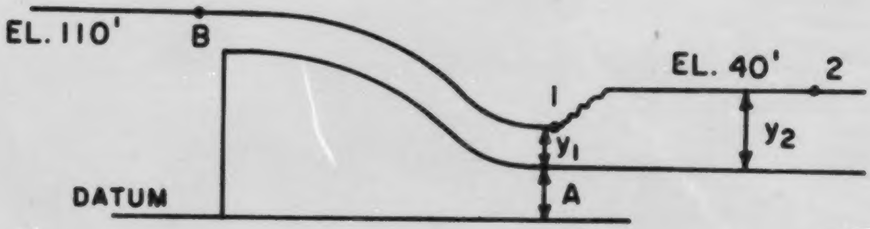
$778 \, \frac{\text{ft-lb}}{\text{BTU}}$

$h_i = 9.9 \times 10^{-3} \, \frac{\text{BTU}}{\text{lb}}$

$V_2 = \frac{Q}{A_2} = \frac{600}{20(7)} = 4.29$ fps

5.4.12:12 Discharge $Q = 120 \frac{ft^3}{sec}$ per ft.

spillway losses = $8 \frac{ft\text{-}lb}{lb}$



1.) Energy equation from B → 2

$$\frac{P_B}{\gamma} + z_B + \frac{V_B^2}{2g} = \frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + L_{B-1}$$

$$\frac{P_B}{\gamma} = 0 \quad , \quad \frac{P_1}{\gamma} = 0$$

$$110 + 10 = (y_1 + A) + \frac{V_1^2}{2g} + 8$$

$$102 = y_1 + A + \frac{V_1^2}{2g}$$

2.) Continuity equation from 1 → 2

$$V_1 = \frac{120}{y_1} \qquad V_2 = \frac{120}{y_2}$$

$$V_1 y_1 = V_2 y_2$$

3.) Hydraulic jump equation

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2V_1^2 y_1}{g}}$$

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2(120)^2 y_1}{y_1^2 g}}$$

$$= -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{894.4}{y_1}}$$

4.) $(y_2 + A) = 40$

solving:

$$102 = y_1 + (40 - y_2) + \frac{(120)^2}{y_1^2(2g)}$$

$$102 = y_1 + 40 - y_2 + \frac{223.6}{y_1^2}$$

$$62 = y_1 - y_2 + \frac{223.6}{y_1^2}$$

$$62 = y_1 - (-\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{894.4}{y_1}} + \frac{223.6}{(y_1)^2}$$

$$62 = (1.5)y_1 - \sqrt{\frac{(y_1)^2}{4} + \frac{894.4}{y_1}} + \frac{223.6}{(y_1)^2}$$

$y_1 = 1.6$ $y_2 = 22.8$ (approximate)

from which $A = 17.1$ ft.

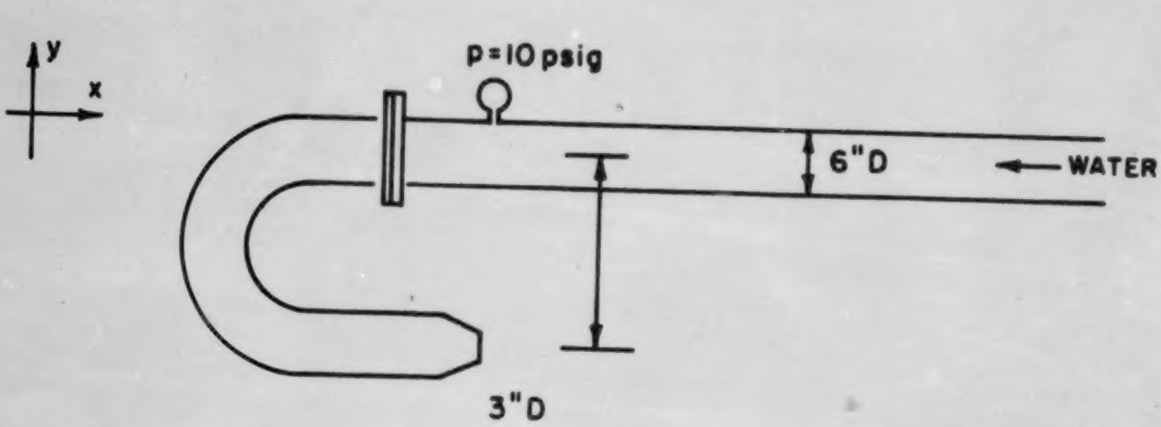
RESOURCE MATERIAL

Ref. No	Chapter	Pages	Other Description
1	3	140-166	Article 3.11

7

QUIZ

1.) The 180° bend is attached to the pipeline by the flanges shown. Compute the x and y components of the force which the flange bolts must resist. Note the xy plane is horizontal.

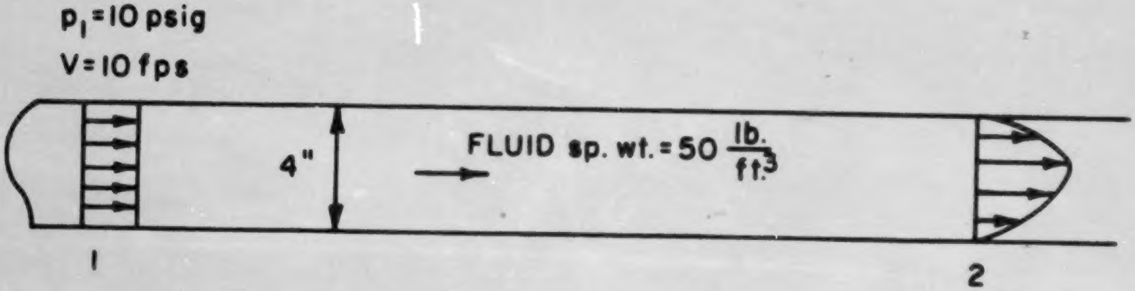


2.) A viscous fluid flows through the pipeline shown. At section 1 the velocity profile is flat and at section 2 the velocity profile is parabolic. The kinetic energy correction factor is 2 and the momentum correction factor is 1.33 (at section 2). Energy losses between 1 and 2 are 10 $\frac{\text{ft-lb}}{\text{lb}}$.

a.) Compute the mean velocity at section 2.

b.) Compute the pressure at section 2.

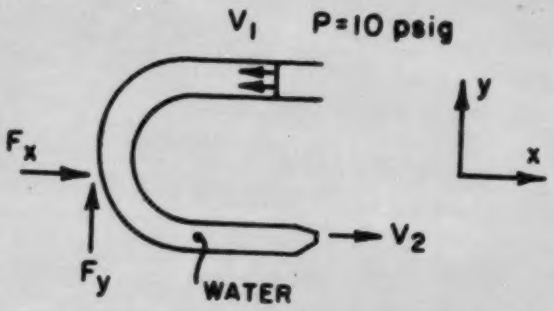
c.) Using the momentum equation, compute the shear force that the fluid exerts on the pipe due to the flow.



QUIZ ANSWERS

1.) $A_1 = \frac{28.27}{144}\ ft^2 = .196\ ft^2$

$A_2 = \frac{7.07}{144} = .049\ ft^2$



Energy equation from 1 → 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\frac{10(144)}{62.4} + \frac{V_1^2}{2g} = \frac{16V_1^2}{2g}$$

$$\frac{10(144)}{62.3} = \frac{15V_1^2}{2g}$$

$$V_1D_1^2 = V_2D_2^2$$

$$V_1(36) = V_2(9)$$

$$V_1(4) = V_2$$

$$V_1 = \sqrt{\frac{1440}{62.4} \times \frac{64.4}{15}} = 9.95 \text{ fps}$$

$$V_2 = 39.8 \text{ fps}$$

Momentum equation for control volume shown

$$\Sigma F_x = \rho Q\ (V_{2x} - V_{1x})$$

$$F_x - P_1A_1 = 1.94(9.95\ A_1)\ [V_2-(-9.95)]$$

$$F_x - P_1A_1 = 1.94(9.95)(.196)\ [39.8 + 9.95]$$

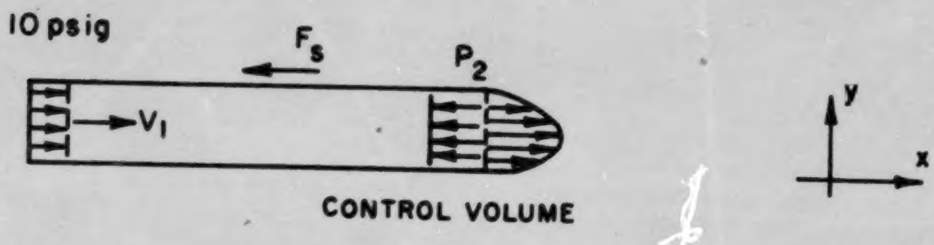
$$F_x = 10(144)(.196) + 1.94(9.95)(.196)\ [49.75]$$

$F_x = 282 + 188.2 = 470.4$ lb. direction correct - bolts would be in tension from F_x.

$$\Sigma F_y = \rho Q\,(V_{2y} - V_{1y}) = 0$$

No force in the y direction, $\therefore\ F_y = 0$

2.)



Energy equation between 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^{\,2}}{2g} = z_2 + L_{1-2}$$

Continuity equation from 1 → 2 $\qquad V_1A_1 = V_2A_2$

since $A_1 = A_2 \ \therefore\ V_1 = V_2$

$$\frac{10(144)}{50} + \frac{(10)^2}{2g} = \frac{P_2}{50} + \frac{2(100)}{2g} + 10$$

$$\frac{144}{5} + \frac{100}{2g} - \frac{200}{2g} - 10 = \frac{P_2}{50}$$

$$28.8 - 1.55 - 10 = \frac{P_2}{50} \qquad ;\ 17.25 = \frac{P_2}{50} \text{ or } P_2 = 5.99 \text{ psig}$$

Momentum equation

$$P_1A_1 - P_2A_2 - F_{shear} = \rho Q\,[\beta_2 V_{2x} - \beta_1 V_{1x}]$$

$$(P_1 - P_2)\,A - F_{shear} = \frac{50}{32.2}\,(10)\,A\,[1.33(10) - 1(10)]$$

$(144)(4.01)(.087) - F_s = \frac{50}{32.2}(100)(.087)(.33) = 4.46$

$50.23 - 4.46 = F_s$; $F_s = 45.8$ lb shear force

(direction O.K. since sign is positive

Answers

a). same as at section 1, 10 fps

b.) 5.99 psig

c.) 45.8 lb →

MODULE 6

SIMILITUDE

SUBJECT MATTER

With the three basic equations of fluid dynamics and the experience gained in their application, you are now in a position to investigate a wide range of engineering problems. However, before you do, the additional concept of similitude will be discussed since it will provide you with an understanding of the technique of data formulation and presentation as well as the concept of modeling. These ideas will be useful in the remainder of your study of Fluid Mechanics.

A simple example of the usefulness of proper data presentation is as follows. Suppose you were interested in determining the drag force F on a smooth sphere of diameter D moving through a viscous incompressible fluid (density ρ and viscosity μ) with velocity V. The drag force is some function of these variables

$$F = F(D, \rho, \mu, V)$$

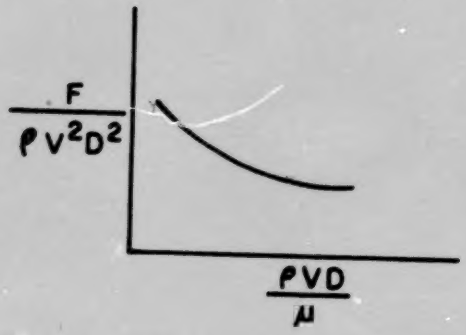
If a mathematical formulation was not available (or if you had derived one and needed an experimental verification) how would you conduct the experiments and present the results for proper interpretation? With 5 variables and only 2 dimensional graph paper this would indeed be difficult if, for example, you used several different fluids, various sphere diameters and a wide range of velocities.

Use an alternate approach. By the methods that you shall learn in this module, the 5 variables can be formulated into two dimensionless groups: namely,

$$\frac{F}{\rho V^2 D^2} \quad \text{and} \quad \frac{\rho V D}{\mu}$$

In addition, it can be shown that $\left(\frac{F}{\rho V^2 D^2}\right) = g\,\frac{(\rho V D)}{\mu}$

where the nature of the functional relationship must come from experimentation. Amazingly, if you take the range of data suggested on page 1 for the variables and plot on linear coordinates for the dimensionless groups shown, a single curve will result! The utility of this alternative procedure is immediately obvious.



Thus one of the purposes of this module is to teach you the technique, called Buckingham Pi Theorem for obtaining these dimensionless groups.

The second purpose of this module is to present the criteria for modeling. Modeling is of fundamental importance to the fluids engineer (can you think of some examples of hydraulic models?) and the question "does the model behave as the prototype" must always be answered before the experimental results can be considered valid and scaled up (or down) for design purposes. If a model and prototype are similar (geometrically, kinematically and dynamically) then the scaling can be done with confidence. The quantitative definition of similarity then is the second purpose of this module.

INCREMENTAL OBJECTIVES

At the conclusion of this module you should be able to:

6.1) Given a set of variables which govern a particular flow problem, formulate these parameters into the proper number of dimensionless groups using the Buckingham Pi Theorem.

6.2) From recall, define (verbally and mathematically) the following dimensionless groups: Weber number, Euler number, Reynolds number, Froude number, and Mach number.

6.3) From recall, define the following: geometric similarity, kinematic similarity, and dynamic similarity.

6.4) Given a model and prototype, compute conditions which must exist for similarity.

PROBLEM INDEX

Problem Number	Problem Description
6.1.1:3	Dimensional analysis (Buckingham Pi Theorem)
6.1.2:3	Dimensional analysis (Buckingham Pi Theorem)
6.1.3:3	Dimensional analysis (Buckingham Pi Theorem)
6.4.1:3	Modeling compressible external flow
6.4.2:3	Modeling internal compressible flow
6.4.3:3	Modeling of hydraulic jump

PROBLEM SET #6

6.1.1:3 Derive an expression for the head lost in an established incompressible flow in a smooth pipe if this loss of head depends only upon diameter and length of pipe, density, viscosity, and mean velocity of the fluid, and acceleration due to gravity.

6.1.2:3 Derive an expression for the drag force on a smooth object moving through compressible fluid if this force depends only upon speed and size of object, and viscosity, density, and modulus of elasticity of the fluid.

6.1.3:3 A circular disk of diameter d and of negligible thickness is rotated at a constant angular speed, ω, in a cylindrical casing filled with a liquid of viscosity μ and density ρ. The casing has an internal diameter D, and there is a clearance y between the surfaces of disk and casing. Derive an expression for the torque required to maintain this speed if it depends only upon the foregoing variables.

6.4.1:3 An airplane wing of 10-ft chord length moves through still air at 59°F and 14.7 psia at a speed of 200 mph. A 1:20 scale model of this wing is placed in a wind tunnel, and dynamic similarity between model and prototype is desired. (a) What velocity is necessary in a tunnel where the air has the same pressure and temperature as that in flight? (b) What velocity is necessary in a variable-density wind tunnel where pressure is 200 psia and temperature is 59°F? (c) At what speed must the model move through water (60°F) for dynamic similarity?

6.4.2:3 When castor oil flows at 60°F in a 2-in. horizontal pipeline at 10 fps, a pressure drop of 480 psi occurs in 200 ft. of pipe. Calculate the pressure drop in the corresponding length of 1-in. pipe when linseed oil at 100°F flows therein at the same Reynolds number.

6.4.3:3 A hydraulic jump from 2 ft to 5 ft is to be modeled in a laboratory channel at a scale of 1:10. What (two-dimensional) flowrate should be used in the laboratory channel? What are the Froude numbers upstream and downstream from the jump in model and prototype?

PROBLEM SET #6--ANSWERS

6.1.1:3 $h_L = f(D, L, \rho, \mu, V, g)$

$n = 7$

$r = \underline{3}$

4Π groups

choose variables:

ρVD as repeating

$\Pi_1 = \rho^{x_1} V^{y_1} D^{z_1} h_L$

$\Pi_2 = \rho^{x_2} V^{y_2} D^{z_2} L$

$\Pi_3 = \rho^{x_3} V^{y_3} D^{z_3} \mu$

$\Pi_4 = \rho^{x_4} V^{y_4} D^{z_4} g$

Dimensionally	
ρ	$\frac{FT^2}{L^4}$
V	L/T
D	L
L	L
h_L	L
μ	$\frac{FT}{L^2}$
g	L/T^2

$\Pi_1 = \frac{h_L}{D}$ by inspection

$\Pi_2 = \frac{L}{D}$ by inspection

$\Pi_3 = \left(\frac{FT^2}{L^4}\right)^{x_3} \left(\frac{L}{T}\right)^{y_3} (L)^{z_3} \quad \frac{FT}{L^2}$

F) $0 = x_3 + 1$ $\qquad x_3 = -1$

L) $0 = -4x_3 + y_3 + z_3 - 2$ $\qquad z_3 = -1$

T) $0 = 2x_3 - y_3 + 1$ $\qquad y_3 = -1$

$\Pi_4 = \left(\frac{FT^2}{L^4}\right)^{x_4} \left(\frac{L}{T}\right)^{y_4} (L)^{z_4} \frac{L}{T^2}$

F) $0 = x_4$

L) $0 = -4x_4 + y_4 + z_4 + 1$; $z_4 = 1$

T) $0 = 2x_4 - y_4 - 2$; $y_4 = -2$

$\Pi_4 = \frac{Dg}{V^2}$

$\Pi_3 = \frac{\mu}{\rho VD}$ or $\frac{\rho VD}{\mu}$

$\Pi_4 = \frac{Dg}{V^2}$ or $\frac{V^2}{Dg}$

$\Pi_1 = f_1\,(\Pi_2, \Pi_3, \Pi_4)$

$h_L = Df_1\,(\frac{L}{D}, \frac{\rho VD}{\mu}, \frac{V^2}{Dg})$

6.1.2:3 Drag Force $F = F\,(V, L, \mu, \rho, K)$

$n = 6$

$r = 3$

Π = 3 groups

Quantity	Dimension
Force	F
Velocity	L/T
Length	L
Viscosity	FT/L^2
Density	FT^2/L^4
K	F/L^2

choose ρVL as repeating variables

$\Pi_1 = \rho^{x_1}\, V^{y_1}\, L^{z_1}\, F$

$\Pi_2 = \rho^{x_2}\, V^{y_2}\, L^{z_2}\, \mu$

$\Pi_3 = \rho^{x_3}\, V^{y_3}\, L^{z_3}\, K$

Π_2 from inspection results in $\frac{\mu}{\rho VL}$ or $\frac{\rho VL}{\mu}$ (Reynolds number)

$\Pi_1 = \rho^{x_1} V^{y_1} L^{z_1} F$

$F \left(\frac{FT^2}{L^4}\right)^{x_1} \left(\frac{L}{T}\right)^{y_1} L^{z_1} F$

F) $0 = x_1 + 1$; $x_1 = -1$

L) $0 = -4x_1 + y_1 + z_1$

T) $0 = 2x_1 - y_1$; $y_1 = -2$

$z_1 = 4x_1 - y_1 = -4 + 2 = -2$

$\Pi_1 = \frac{F}{\rho V^2 L^2}$

$\Pi_3 = \rho^{x_3} V^{y_3} L^{z_3} K$

$\Pi_3 = \left(\frac{FT^2}{L^4}\right)^{x_3} \left(\frac{L}{T}\right)^{y_3} (L)^{z_3} \frac{F}{L^2}$

F) $0 = x_3 + 1$; $x_3 = -1$

L) $0 = -4x_3 + y_3 + z_3 - 2$ and

$0 = +4 - 2 + z_3 - 2$

$z_3 = 0$

T) $0 = 2x_3 - y_3$; $y_3 = -2$

$\Pi_3 = \frac{K}{\rho V^2}$

$$\frac{F}{\rho V^2 L^2} = g\left(\frac{\rho VL}{\mu}, \frac{V^2}{K/\rho}\right)$$

$$F = \rho V^2 L^2 g\left(\frac{\rho VL}{\mu}, \frac{V^2}{K/\rho}\right)$$

6.1.3:3 Torque $T = f(d, \omega, \mu, \rho, D, y)$

or $g(T, d, \omega, \mu, \rho, D, y) = 0$

$n = 7$

$r = 3$

4 Π groups

choose

d, ω, ρ as repeaters

$\Pi_1 = d^{x_1} \omega^{y_1} \rho^{z_1} T$

$\Pi_2 = d^{x_2} \omega^{y_2} \rho^{z_2} \mu$

$\Pi_3 = d^{x_3} \omega^{y_3} \rho^{z_3} D$

$\Pi_4 = d^{x_4} \omega^{y_4} \rho^{z_4} y$

Quantity	Dimension
torque	FL
diameter	L
angular velocity	1/T
viscosity	FT/L^2
density	FT^2/L^4
diameter	L
clearance	L

By inspection, $\Pi_2 = \dfrac{\mu}{d^2\omega\rho}$

$\Pi_3 = \dfrac{D}{d}$, $\Pi_4 = \dfrac{y}{d}$

$$\Pi_1 = d^{x_1} \omega^{y_1} \rho^{z_1} T$$

$$= L^{x_1} \left(\frac{1}{T}\right)^{y_1} \left(\frac{FT^2}{L^4}\right)^{z_1} FL$$

F) $0 = z_1 + 1$; $z_1 = -1$

L) $0 = x_1 - 4z_1 + 1$; $x_1 = -5$

T) $0 = -y_1 + 2z_1$; $y_1 = -2$

$\Pi_1 = \dfrac{T}{d^5\omega^5\rho}$ Thus $\dfrac{T}{\rho\omega^2 d^5} = f\left(\dfrac{\omega d^2\rho}{\mu}, \dfrac{D}{d}, \dfrac{y}{d}\right)$

6.4.1:3 Airplane wing $P = 14.7$ psia

chord length = 10 ft. $V = 200$ mph

$T = 59^\circ F.$

Dynamic similarity assured for constant Reynolds number

$$Re_m = Re_p$$

a) $$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$V_m = \frac{L_p}{L_m} \cdot V_p \quad \text{since } \nu_n = \nu_p$$

$$V_m = \frac{20}{1}(200) = 4000 \text{ mph! unrealistic}$$

b) $$\frac{V_m L_m \rho_m}{\mu_m} = \frac{V_p L_p}{\nu_p}$$ $$\nu = 1.7 \times 10^{-4} \frac{ft^2}{sec} (\text{for } 14.7 \text{ psia, } 59^\circ F)$$

$$\frac{V_m (1)(.032)}{3.9 \times 10^{-7}} = \frac{200(20)}{1.7 \times 10^{-4}}$$ $$\mu_{59^\circ} = 3.9 \times 10^{-7} \frac{lb\text{-}sec}{ft^2}$$

$$V_m = 200\left(\frac{20}{1}\right) \frac{3.9 \times 10^{-7}}{1.7 \times 10^{-4}} \frac{1}{(.032)}$$ $$\rho = \frac{P}{RT} = \frac{200(144)}{53.3(519)} = 1.04$$

$$\rho = .032 \text{ slugs/ft}^3$$

$$= 286.7 \text{ mph}$$

c) $$\frac{V_m (1)}{1.217 \times 10^{-5}} = \frac{200\ (20)}{1.7 \times 10^{-4}}$$

$$V_m = 200\ (20) \frac{(1.217)}{(1.7)} \times 10^{-1} = 286.3 \text{ mph}$$

6.4.2:3 castor oil at $60^\circ F$ 2" horizontal pipe

$V = 10$ fps

$\Delta P = 480$ psi

$L = 200$ ft

linseed oil

$\mu = 51 \times 10^{-5} \frac{\text{lb-sec}}{\text{ft}^2}$

sp. gr. = .94

castor oil

$\mu = 2900 \times 10^{-5} \frac{\text{lb-sec}}{\text{ft}^2}$

sp. gr. = .96

1" pipe

linseed oil @ $100^\circ F$

$Re_L = Re_{co}$

$$\frac{V_L D_L \rho_L}{\mu_L} = \frac{V_{co} D_{co} \rho_{co}}{\mu_{co}}$$

$$\frac{V_L (1)(.94)\rho_w}{51 \times 10^{-5}} = \frac{V_{co}(2)(.96)\rho_w}{2900 \times 10^{-5}}$$

$$V_L = 10\left(\frac{2}{1}\right)\left(\frac{.96}{.94}\right)\frac{51}{2900}$$

$$V_L = 20\left(\frac{.96}{.94}\right)\frac{51}{2900} = 0.359 \text{ ft/sec}$$

$$\left.\frac{\Delta P}{\rho V^2}\right)_L = \left.\frac{\Delta P}{\rho V^2}\right)_{co}$$

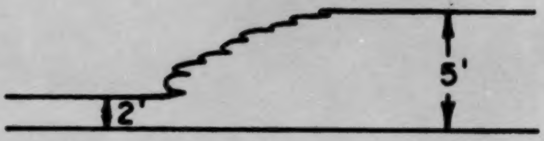
$$\Delta P_L = \Delta P_{co} \frac{(\rho V^2)_L}{(\rho V^2)_{co}} = \Delta P \left(\frac{.94}{.96}\right)\left(\frac{.359}{10}\right)^2$$

$$\Delta P_L = 1.26 \times 10^{-3} \Delta P_{co}$$

$$= 1.26 \times 10^{-3} (480)$$

$\Delta P_L = .605$ psi

6.4.3:3 Hydraulic Jump



a) $$y_2 = \frac{-y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2V_1^{\,2}y_1}{g}}$$

$$5 = -1 + \sqrt{1 + \frac{2V_1^{\,2}(2)}{g}}$$

$$6 = \sqrt{1 + \frac{4V_1^{\,2}}{g}} , \quad 36 = 1 + \frac{4V_1^{\,2}}{g} , \quad 35 = \frac{4V_1^{\,2}}{g}$$

$$V_1 = \sqrt{\frac{35g}{4}}$$

$$= 16.78 \text{ fps}$$

b) $$F_{R_m} = F_{R_p}$$

$$\left.\frac{V_1^{\,2}}{gy_1}\right)_m = \left.\frac{V_1^{\,2}}{gy_1}\right)_p$$

$$V_{1_m}^{\,2} = V_{1_p}^{\,2}\left(\frac{y_{1_m}}{y_{1_p}}\right) = \left(\frac{1}{10}\right)(16.78)^2$$

$V_1^{\,2} = 28.175$, $V_1 = 5.31$ fps for a 1/10 model

$$y_1 = \frac{2}{10} = .2 \text{ ft}$$

$Q = (.2)(5.31) = 1.06$ ft^{3}sec-ft

Froude number same upstream and downstream in model and prototype

$$5V_2 = 2V_1 , \quad V_2 = \frac{2}{5}V_1$$

$$= 6.71 \text{ fps}$$

$$F_{R_1} = \frac{V^2}{gL} = \frac{(16.78)^2}{(32.2)(2)} = 4.372 \qquad F_{R_2} = \frac{(6.71)^2}{(32.2)(5)} = .279$$

RESOURCE MATERIAL

Ref. No.	Chapter	Pages	Other Description
1	4	201-225	Articles 4.1-4.5
7			Similitude
8			Dimensional Analysis and theory of modeling

QUIZ

Directions: Problems 1-10 count 8 points each. Fill in the blanks to make the problems read correctly. Problem 11 counts 20 points.

1.) The Weber number defines the ratio of __________ forces to __________ forces and may be expressed as ____________ where (define symbols)

____.

2.) The Mach number defines the ratio of __________ forces to __________ forces and may be expressed as ____________ where (define symbols)

____.

3.) The Euler number defines the ratio of __________ forces to __________ forces and may be expressed as __________ where (define symbols)

____.

4.) The Reynolds number defines the ratio of __________ forces to __________ forces and may be expressed as ____________ where (define symbols)

____.

5.) The Froude number defines the ratio of __________ forces to __________ forces and may be expressed as ____________ where ______________

____.

6.) The power P required to drive a propeller is known to depend on

D = propeller diameter

ρ = density of fluid

c = velocity of sound

ω = angular velocity of propeller

V = free stream velocity

μ = viscosity

How many Π groups characterize this problem? __________

7.) A valid set of repeating variables for Problem 6 would be

____.

8.) For Problem 6, show the solution for one of the Π groups.

9.) State the conditions which must exist for complete similarity to exist between a model and its prototype (briefly explain).

____

____

____

10.) An open cylindrical tank of radius r_o is rotated about its central axis at a constant rate of ω rpm. After a time, this rotation produces a constant difference in water surface elevation between the tank wall and the axis of rotation (h). If this problem [illegible] to be modeled, what forces are required for dynamic similarity?

11.) The drag of a small submarine hull is desired when it is moving far below the free surface of water. A 1/10 model is to be used in water. If the drag of the prototype at 1.15 mph is desired, at what speed should the model be tested?

QUIZ ANSWERS

Directions: Problems 1-10 count 8 points each. Fill in the blanks to make the problems read correctly. Problem 11 counts 20 points.

1.) The Weber number defines the ratio of surface tension forces to inertia forces and may be expressed as $\frac{\rho V^2 L}{\sigma}$ where (define symbols)

V = mean velocity, ρ = mass density, L = length parameter, σ = surface tension coefficient.

2.) The Mach number defines the ratio of elastic forces to inertia forces and may be expressed as V/C where (define symbols)

V = mean velocity ; C = sonic velocity.

3.) The Euler number defines the ratio of pressure forces to inertia forces and may be expressed as $\frac{\Delta P}{\rho V^2}$ where (define symbols)

ΔP = pressure drop, ρ = mass density, V = mean velocity.

4.) The Reynolds number defines the ratio of inertia forces to viscous forces and may be expressed as $\frac{\rho VD}{\mu}$ where (define symbols)

ρ = mass density, V = mean velocity, D = length parameter, μ = absolute viscosity.

5.) The Froude number defines the ratio of inertia forces to gravity forces and may be expressed as $\frac{V^2}{gy}$ where

V = mean velocity, g = acceleration due to gravity, y = length parameter (usually flow depth).

6) The power P required to drive a propeller is known to depend on

D = propeller diameter

ρ = density of fluid

C = velocity of sound

ω = angular velocity of propeller

V = free stream velocity

μ = viscosity

How many Π groups characterize this problem? 4

7.) A valid set of repeating variables for Problem 6 would be

ω, D, ρ.

8.) For Problem 6, show the solution for one of the Π groups.

$$\Pi_1 = \rho^{x_1} D^{y_1} \omega^{z_1} V \qquad \rho \rightarrow \frac{M}{L^3}$$

$$D \rightarrow L$$

$$\Pi_1 = \left(\frac{M}{L^3}\right)^{x_1} (L)^{y_1} \left(\frac{1}{T}\right)^{z_1} \left(\frac{L}{T}\right) \qquad \omega \rightarrow \frac{1}{T}$$

$$M^0L^0T^0 = \left(\frac{M}{L^3}\right)^{x_1} (L)^{y_1} \left(\frac{1}{T}\right)^{z_1} \left(\frac{L}{T}\right) \qquad V \rightarrow \frac{L}{T}$$

M) $0 = x_1$ $\qquad x_1 = 0$

L) $0 = -3x_1 + y_1 + 1$ $\qquad y_1 = -1 \longrightarrow \Pi_1 = \frac{V}{D\omega}$

T) $0 = -z_1 - 1$ $\qquad z_1 = -1$

9.) State the conditions which must exist for complete similarity to exist between a model and its prototype (briefly explain).

1.) Geometric Similarity - All length ratios must be the same

2.) Kinematic Similarity - All velocities and accelerations at corresponding points must be in the same ratio.

3.) Dynamic Similarity - All forces (individually and collectively) must be in the same ratio.

10.) An open cylindrical tank of radius r_o is rotated about its central axis at a constant rate of ω rpm. After a time, this rotation produces a constant difference in water-surface elevation between the tank wall and the axis of rotation (h). If this problem were to be modeled, what forces are required for dynamic similarity?

Inertia forces, gravity forces

In other words, Froude number must be the same in model and prototype.

11.) The drag of a small submarine hull is desired when it is moving far below the free surface of water. A $\frac{1}{10}$ model is to be used in water. If the drag of the prototype at 1.15 mph is desired, at what speed should the model be tested?

$\underline{Re_m = Re_p}$ for dynamic similarity

$$\left.\frac{VD}{\nu}\right|_m \quad \left.\frac{VD}{\nu}\right|_p$$

$$\nu_m = \nu_p$$

$$V_m = V_p \left(\frac{D_p}{D_m}\right) = V_p \left(\frac{10}{1}\right)$$

$$V_m = 11.5 \text{ mph}$$

MODULE 7

VISCOUS FLUID FLOW

SUBJECT MATTER

For simplicity, many of the problems considered in Modules 4 and 5 involved ideal fluids, i.e., incompressible nonviscous fluids. If fluid resistance was present, a head loss term was included in the energy equation. Although useful, this approach is unsatisfactory from your point of view since it does not indicate the fundamental nature of fluid resistance which is required for understanding and subsequent applications.

In Module 6, the Reynolds number was found to express the ratio of inertia forces to viscous forces. In this module, you shall see that for low values of the Reynolds number (relatively low inertia forces, high viscous forces) the flow is streamlined or laminar. The exact range of Reynolds number for laminar flow to exist is not constant but depends on the geometry of the flow (i.e., flow in a pipe, flow in a channel or flow over a sphere, etc). At high Reynolds number (high inertia forces, low viscous forces) the streamlined flow breaks down and a chaotic, randomly fluctuating flow, called turbulent flow, exists. Your concept of the word turbulence will give you an intuitive idea of the nature of this type flow. If you look at a point in space, the velocity pressure, temperature, etc., fluctuate about a mean value. Thus the instantaneous velocity components could be expressed as

x component $u = \bar{u} + u'$

y component $v = \bar{v} + v'$

z component $w = \bar{w} + w'$

where in each case the first term is the mean value and the second term the fluctuating component.

In terms of analysis, laminar flow is well defined mathematically.

Thus for a great many problems analytical solutions exist and the laboratory substantiation of these solutions is excellent. Turbulent flow, however, is quite different. The complex nature of the flow precludes a simple, straightforward constitutive equation(i.e., stress-rate of strain relationship) like Newton's law of viscosity. Without considering a stochastic model,you have no recourse but to use empirical relationships. Some of the more common empirical equations will be discussed in this module. Applications of the basic equations to external flows (flow over cylinders and spheres, flat plate boundary layer) and internal flows (pipe flow, open channel flow) will be considered in Module 8.

INCREMENTAL OBJECTIVES

At the conclusion of this module, you should be able to:

7.1) Define laminar and turbulent flow including the fundamental differences in flow characteristics, resistance and Reynolds number.

7.2) For laminar flow, set up the basic force balance on an element (which fits the geometry under study). Integrate the resulting equation to obtain the velocity profile using Newton's law of viscosity relating the shear stress to the rate of strain.

7.3) For laminar flow, set up the basic differential equation for power dissipation of an element. Using Newton's law of viscosity, integrate the basic equation to obtain an expression for the total power dissipation.

7.4) For turbulent flow, set up the basic force balance on an appropriate element. Using Prandtl's mixing length, integrate the resulting equation to obtain the velocity profile for a pipe and for a smooth plane surface.

7.5) For turbulent flow, convert the basic velocity profile into the velocity defect expression by suitably defining all constants.

PROBLEM INDEX

Problem Number	Problem Description
7.1.1:4 (7.3.1:1)	Computation of energy dissipation in laminar pipe flow
7.1.2:4	Laminar pipe flow
7.1.3:4	Evaluation of mean velocity using the Reynolds number
7.1.4:4	Prandtl mixing length in turbulent flow
7.2.1:3	Derivation of velocity profile in two dimensional laminar flow
7.2.2:3	Application of laminar flow
7.2.3:3	Derivation of velocity profile for laminar flow of liquid down an inclined plane
7.4.1:2	Turbulent pipe flow
7.4.2:2	Turbulent pipe flow

PROBLEM SET #7

7.1.1:4 (7.3.1:1)

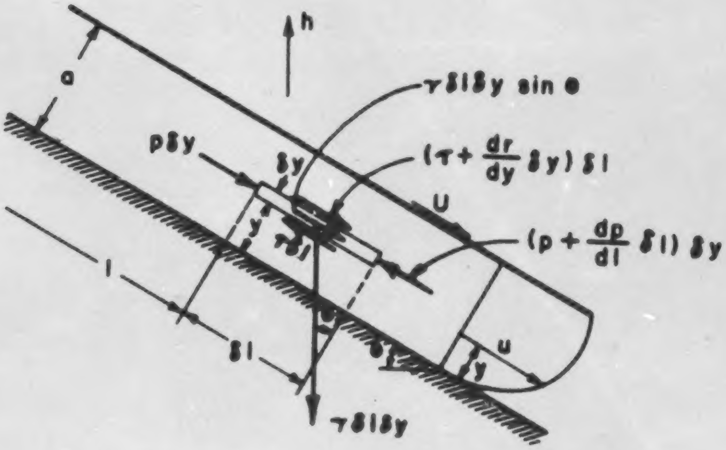
Determine the pressure drop per 100 feet of $\frac{1}{8}$ inch ID tubing for flow of liquid, $\mu = 60$ centipoises, sp. gr. = 0.83 at a Reynolds number of 100.

7.1.2:4 Glycerin at 80°F flows through a $\frac{3}{8}$ inch diameter pipe with a pressure drop of 5 psi/ft. Find the discharge and the Reynolds number.

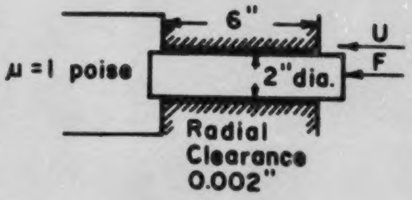
7.1.3:4 Calculate the flow of crude oil, sp. gr. 0.86, at 80°F in a $\frac{3}{8}$ inch diameter tube to yield a Reynolds number of 1200.

7.1.4:4 By use of the one-seventh-power law of velocity distribution $\frac{u}{u_{max}} = (y/r_o)^{1/7}$, determine the mixing-length distribution L/r_o in terms of y/r_o from the equation, $L = K \frac{du/dy}{d^2u/dy^2}$

7.2.1:3 Determine the formulas for shear stress on each plate and for the velocity distribution for flow in the figure below when an adverse pressure gradient exists such that Q = 0.


h
a
$\gamma \delta l \delta y \sin\theta$
$p\delta y$
δy
$(\tau + \frac{d\tau}{dy}\delta y)\delta l$
U
$(p + \frac{dp}{dl}\delta l)\delta y$
y
$\tau\delta l$
l
θ
δl
u
y
δl
$\gamma \delta l \delta y$

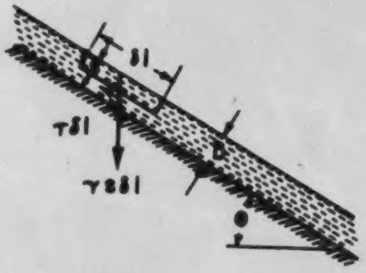
7.2.2:3 In the figure below, U = 2 ft/sec. Find the rate at which oil is carried into the pressure chamber by the piston and the shear force and total force F acting.


6"
U
F
μ =1 poise
2"dia.
Radial
Clearance
0.002"

7.2.3:3 With a free body, as in the figure below, for uniform flow of a thin laminar of liquid down an inclined plane, show that the velocity distribution is

$u = \frac{\gamma}{2\mu}(b^2 - S^2) \sin \Theta$ and that the discharge per unit width is

$Q = \frac{\gamma}{3\mu} b^3 \sin \Theta$


δl
τδl
γsδl
b
θ

7.4.1:2 Find the value of y/r_o in a pipe where the velocity equals the average velocity for turbulent flow.

7.4.2:2 Plot the velocity profiles for Prandtl's exponential velocity formula for values of n of $\frac{1}{7}$, $\frac{1}{8}$, and $\frac{1}{9}$.

PROBLEM SET #7--ANSWERS

7.1.1:4 (7.3.1:1)

$L = 100$ feet $\qquad \mu = 60$ centipoise $= .6$ poise

$D = \frac{1}{8}$ inch ID $\qquad$ sp. gr. $= .83$

$Re = 100 \qquad \rho = .83\,(1.94) = 1.61$ slugs/ft^3

Flow is obviously laminar

$$\mu = \frac{.6}{479}\ \frac{\text{lb-sec}}{\text{ft}^2} = 1.25 \times 10^{-3}\ \frac{\text{lb-sec}}{\text{ft}^2}$$

For laminar pipe flow

$$\Delta P = \frac{32\ \mu L V}{D^2} \quad ; \quad Re = \frac{VD\rho}{\mu}$$

$$100 = \frac{V\ \frac{1}{8(12)}(1.61)}{1.25} \times 10^3$$

$$V = \frac{(.1)(1.25)(96)}{1.61} = 7.45 \text{ ft/sec}$$

$$\Delta P = \frac{32(1.25)\ 10^{-3}\ (100)(7.45)}{\left(\frac{1}{8(12)}\right)^2}$$

$$\Delta P = [32\ (1.25)\ 100(7.45)(96)^2] \times 10^{-3}$$

$$= 2.75 \times 10^5 \text{ lb/ft}^2$$

$$= 1907 \text{ lb/in}^2$$

7.1.2:4 glycerin @ $80^{\circ}F$

$D = \frac{3}{8}$ inch diameter

$$\Delta P = 5 \frac{\text{lb-in}^2}{\text{ft}}$$

From figure C. 2 in Reference #1

$\nu = 3 \times 10^{-3}\ \text{ft}^2/\text{sec}$

$$\mu = 1 \times 10^{-2} \frac{\text{lb-sec}}{\text{ft}^2}$$

assume laminar flow

$$\Delta P = \frac{32\ \mu L V}{D^2}$$

$$\frac{\Delta P}{L} = 5(144) = \frac{32\ \mu V}{D^2}$$

$$5(144) = \frac{32(10^{-2})}{(\frac{3}{8(12)})^2} V = \frac{32 \times 10^{-2}\ V}{(\frac{1}{32})^2}$$

$$\frac{5(144)}{(32)^2}\frac{(10)^2}{(32)} = V = 2.197\ \text{ft/sec}$$

check $\quad Re = \frac{VD}{\nu} = \frac{2.197\ [(\frac{3}{8})(12)]}{3 \times 10^{-3}}$

$$Re = \frac{2.197}{96}(1000) = 22.89$$

laminar flow, therefore, assumption O.K.

volumetric flow = $Q = VA$

$$= 2.197 \times \frac{\Pi}{4}[(\frac{3}{8})(12)]^2$$

$$= 2.197 \times \frac{\Pi}{4}(\frac{1}{32})^2$$

$$= .00168\ \text{ft}^3/\text{sec}$$

7.1.3:4 sp. gr. = .86

$$D = \frac{3''}{8}$$

$$Re = 1200$$

$$\rho = 1.663 \text{ slugs/ft}^3$$

$$\nu = 8.5 \times 10^{-5} \text{ ft}^2\text{/sec}$$

$$Re = \frac{VD}{\nu}$$

$$1200 = \frac{V \frac{3}{8(12)}}{8.5} \times 10^5$$

$$V = \frac{1200(8.5)(32)}{10^5} = 3.264 \text{ ft/sec (mean velocity)}$$

$$Q = VA$$

$$= 3.264 \frac{\Pi}{4} \left(\frac{3}{8(12)}\right)^2$$

$$= 3.264 \frac{\Pi}{4} \left(\frac{1}{32}\right)^2$$

$$= 0.00250 \text{ ft}^3\text{/sec}$$

7.1.4:4 $u/u_{max} = (y/r_o)^{1/7}$

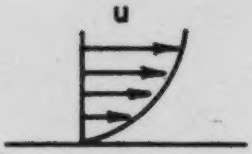
mixing length L/r_o in terms of y/r_o.

$$L = K \left| \frac{du/dy}{\frac{d^2\mu}{dy^2}} \right|$$

where $\frac{du}{dy} = \frac{1}{7} y^{-6/7} \frac{1}{r_o^{1/7}}$

$$\frac{dy}{dy} = \frac{1}{7}\,\frac{1}{r_o^{\,1/7}}\;y^{6/7}$$

also $\dfrac{u}{u_{max}} = \left(\dfrac{y}{r_o}\right)^{1/7}$



$$\frac{du}{dy} = \frac{1}{7}\left(\frac{y}{r_o}\right)^{-6/7}\left(\frac{1}{r_o}\right)$$

$$\frac{d^2u}{dy^2} = \frac{1}{7r_o^{\,1/7}}\,\frac{d}{dy}\left(\frac{1}{y^{6/7}}\right)$$

$$= \frac{1}{7r_o^{\,1/7}}\;\frac{(-1\;\frac{6}{7}\,y^{-1/7})}{(y^{6/7})^2}$$

$$= -\,\frac{1}{7r_o^{\,1/7}}\,\frac{(6)}{7}\,\frac{1}{y^{\,1/7}\cdot y^{\,12/7}} = \frac{-6}{49r_o^{\,1/7}}\,\frac{1}{y^{\,13/7}}$$

substituting:

$$L = K\left|\left(\frac{1}{7}\,\frac{1}{r_o^{\,1/7}}\,y^{6/7}\right)\cdot\left(\frac{1}{(\frac{-6}{49})\,\frac{1}{r_o^{\,1/7}}\,\frac{1}{y^{\,13/7}}}\right)\right|$$

$$L = +K\left(\frac{7}{6}\right)y$$

$$\frac{L}{r_o} = -\,\frac{7}{6}K\,\frac{y}{r_o}$$

7.2.1:3 since $Q = 0$

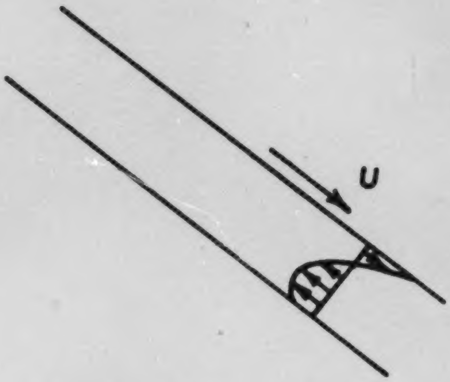
$$Q = \frac{Ua}{2} - \frac{1}{12\mu}\frac{d(p + \gamma h)}{\partial L}(a^3)$$

adverse pressure gradient means $\frac{d(p)}{dl}$ is positive

$$Q = \frac{Ua}{2} - \frac{1}{12\mu}\frac{dp}{dl}a^3 + \frac{1}{12\mu}\sin\theta\,\gamma\,a^3$$

$$\text{or}\quad \frac{Ua}{2} = \frac{1}{12\mu}\frac{dp}{dl}a^3 - \frac{1}{12\mu}\gamma\,a^3\sin\theta$$

$$= \frac{1}{12\mu}\frac{d}{dl}(p + \gamma h)\,a^3$$



Recall

$$u = U\frac{y}{a} - \frac{1}{2\mu}\frac{d}{dl}(p + \gamma h)(ay - y^2)$$

$$u = U\frac{y}{a} - \frac{1}{2\mu}\left(\frac{Ua}{2}\frac{12\mu}{a^3}\right)(ay - y^2)$$

$$u = U\frac{y}{a} - 3\frac{U}{a^2}(ay - y^2)$$ velocity profile (see above)

shear stress

$$\tau = \mu\frac{du}{dy} = \mu\left[\frac{U}{a} - \frac{3U}{a^2}(a - 2y)\right]$$

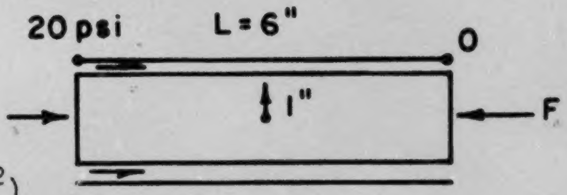
Bottom plate $y = 0$

$$\tau = \mu\left[\frac{U}{a} - \frac{3U}{a}\right] = -2\mu\frac{U}{a}$$

Top plate $y = a$

$$\tau = \mu\left[\frac{U}{a} - \frac{3U}{a^2}(a^2 - a^2)\right] = \mu\frac{U}{a}$$

7.2.2:3 Using the 2-D equations developed in the text of Reference #1, for the Couette flow (valid since clearance is small compared to the radius)



$$\mu = \frac{1}{479} = .00209 \ \frac{\text{lb-sec}}{\text{ft}^2}$$

$$u = U\frac{y}{a} - \frac{1}{2\mu}\frac{d}{dl}(p + \gamma h)(ay - y^2)$$

Here $\frac{dh}{dl} = 0 \quad (\theta = 0^\circ)$

$$u = U\frac{y}{a} - \frac{1}{2\mu}\left(\frac{dp}{dl}\right)(ay - y^2) \quad ; \quad \frac{dp}{dl} = \frac{\Delta P}{\Delta l} = \frac{(20 - 0)(144)}{6/12}$$

$$u = U\frac{y}{a} - \frac{1}{2\mu}[20(288)](ay - y^2)$$

$$Q = \frac{Ua}{2} - \frac{1}{12\mu}\frac{\Delta P}{\Delta l}a^3$$

$$Q = \left[2\frac{(.002)}{12} - \frac{1}{12(.00209)}\frac{20(288)}{}\left(\frac{.002}{12}\right)^3\right] \times \frac{\Pi(1.001)}{12}$$

$$= (3.33 \times 10^{-4}) - 1.53 \times 10^{-4}$$

$$= 1.356 \times 10^{-4} \ \text{ft}^3/\text{sec}$$

shear force on the piston

$$\tau = \mu\frac{du}{dy} = \mu\left[\frac{U}{a} - \frac{1}{2\mu}(20)(288)(a - 2y)\right]_{y = a}$$

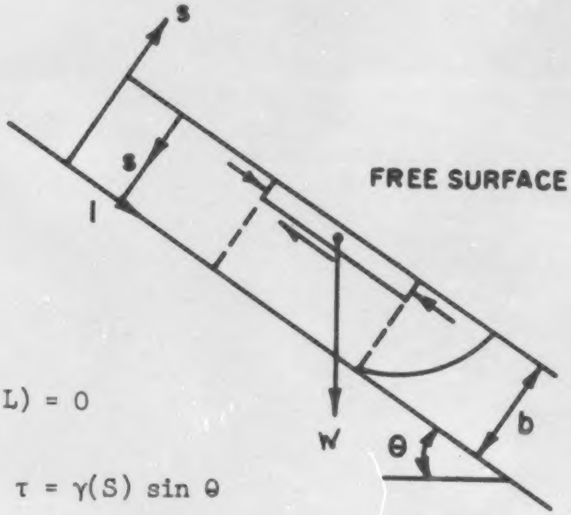
$$= \mu\left[\frac{2(12)}{(.002)} - \frac{1}{2\mu}(20)(288)\left(\frac{-.002}{12}\right)\right]$$

$$= \frac{24}{(.002)}(.00209) + \frac{1}{2}(20)(288)\left(\frac{.002}{12}\right) = 25.5 \ \text{lb/ft}^2$$

Total Force = Δp Area + τA_{shear}

$$F = 20\Pi\,(1)^2 + \frac{25.5}{144}(6)\,\Pi\,(2) = 62.8 + 6.68$$

$$F = 69.5 \text{ lb}$$



7.2.3:3 uniform steady

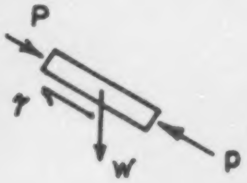
weight - shear = 0

$$\sin\theta\,\gamma\,S(\delta L) - \tau(S)(1)(\delta L) = 0$$

$$\tau = \gamma(S)\sin\theta$$

$$\tau = -\mu\left(\frac{\partial u}{\partial S}\right)$$ no pressure change

$$\tau = -\mu\frac{\partial u}{\partial S} = +\gamma\,S\sin\theta$$



$$\frac{\partial u}{\partial S} = -\frac{1}{\mu}\gamma\,S\sin\theta$$

$$u = -\frac{\gamma}{\mu}\int S\,dS\sin\theta$$

$$u = -\frac{\gamma}{\mu}\cdot\frac{S^2}{2}\sin\theta + C$$

$$0 = -\frac{\gamma}{\mu}\frac{b^2}{2}\sin\theta + C$$

$$C = +\frac{\gamma}{\mu}\cdot\frac{b^2}{2}\sin\theta$$

$$u = \frac{\gamma}{2\mu}(b^2 - S^2)\sin\theta$$

Volumetric flow

$$Q = \int udA$$

$$Q = \int udA = \sin \theta \int_o^b \frac{\gamma}{2\mu} (b^2 - S^2)(1)\, dS$$

$$= \sin \theta \frac{\gamma}{2\mu} (b^2 S - \frac{S^3}{3})_o^b \quad \frac{2}{3} b^3$$

$$= \frac{\gamma}{3\mu} \sin \theta\, b^3$$

7.4.1:2 $\frac{y}{r_o}$ where velocity equals average velocity for turbulent pipe flow

Using the logarithmic form for the velocity profile

$$\frac{u}{u_*} = \frac{u_m}{u_*} + \frac{1}{K} \ln \frac{y}{r_o}$$

mean velocity $V = u_m - \frac{3}{2} \frac{u_*}{K}$

$$\frac{u_m}{u_*} - \frac{3}{2} \frac{1}{K} = \frac{V_m}{V_*} + \frac{1}{K} \ln \frac{y}{r_o}$$

$$- \frac{3}{2K} = \frac{1}{K} \ln \frac{y}{r_o}$$

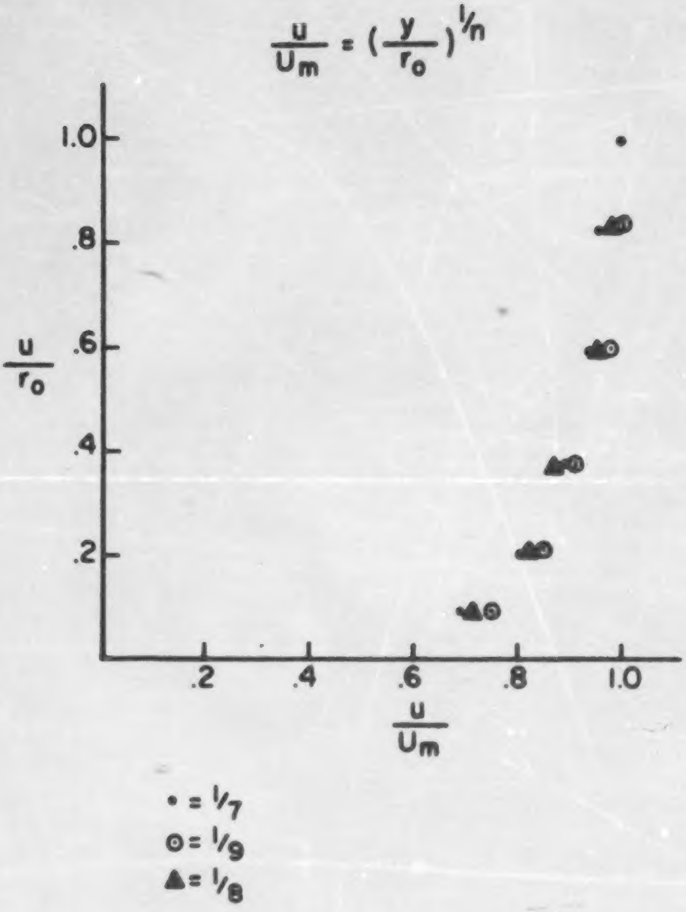
$$- \frac{3}{2} = \ln (\frac{y}{r_o})$$

$$\ln \frac{y}{r_o} = - \frac{3}{2}$$

$$\frac{y}{r_o} = e^{-3/2} = e^{-1.5}$$

$$\frac{y}{r_o} = \frac{1}{e^{1.5}} = .223$$

7.4.2:2



RESOURCE MATERIAL

Ref. No.	Chapter	Pages	Other Description
1	5	233-258	Articles 5.1-5.5
13			FM-8 Film Loop
14			RFM-134 Film Loop
15			RFM-136 Film Loop

QUIZ

1.) Give a qualitative description of laminar flow ______________________

____

____

Mathematically, laminar flow is defined by the stress-rate of strain equation ____________________ where (define symbols) ____________________

____.

2.) Give a qualitative description of turbulent flow ______________________

____

____

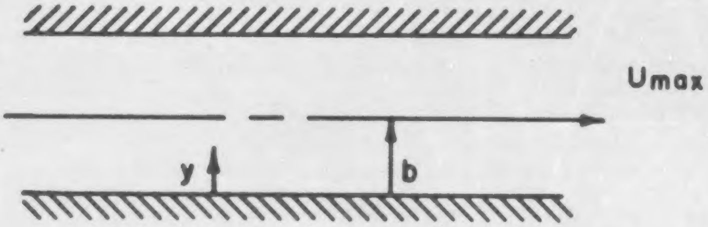
Mathematically, turbulent flow is defined by the stress-rate of strain equation ______________________________ where (define symbols) ______

____.

3.) Glycerin at $100^{\circ}F$ flows through a one inch diameter tube at a flow rate of .03 cfs. Compute the power dissipated by this flow in a 25 foot length of pipe.

4.) Two-dimensional turbulent flow between parallel plates is described by the velocity profile

$$\frac{u}{U_{max}} = \left(\frac{y}{b}\right)^{1/6}$$



Compute the mean velocity in terms of the maximum velocity.

QUIZ ANSWERS

1.) Give a qualitative description of laminar flow:

fluid particles move in well defined parallel paths, flow resistance is a minimum.

Mathematically, laminar flow is defined by the stress-rate of strain equation $\tau = \mu \, du/dy$ where (define symbols)

μ = viscosity, u = velocity, y = distance from solid wall.

2.) Give a qualitative description of turbulent flow:

flow properties randomly fluctuate about some mean value (velocity and pressure). Fluid particles do not move in well defined paths--good mixing of all fluid layers, resistance is greater than for laminar flow.

Mathematically, turbulent flow is defined by the stress-rate of strain equation $\tau = \rho L^2 (d\bar{u}/dy)^2$ where

ρ = density, L = Prandtl's mixing length,

$d\bar{u}/dy$ = velocity gradient of mean velocity profile.

3.) at 100°F, $\nu = 1.45 \times 10^{-3}$ ft^2/sec

$$\mu = 3.9 \times 10^{-3} \ \frac{\text{lb-sec}}{\text{ft}^2}$$

$$V = \frac{.03(144)}{\Pi/4} = 5.5 \text{ fps}$$

$$Re = \frac{VD}{\nu} = \frac{5.5(1)\,10^3}{12(1.45)}$$

Re = 316 , laminar flow

$$\Delta P = \frac{128\ \mu LQ}{\Pi D^4} = \frac{128(3.9)(10^{-3})(25)(.03)(1.2)^4}{\Pi(1)}\ 10^4$$

$$= \frac{128(3.9)(25)(.3)(1.2)^4}{\Pi} = 2471\ \ \text{lb/ft}^2$$

$$\text{Power} = \Delta P\ Q = 2471 \times .03 = 74.1\ \frac{\text{ft-lb}}{\text{sec}}$$

4.) $V_{mean} = \frac{Q}{2b} = \frac{12}{14}\ V_{max} = \frac{6}{7}\ V_{max}$

$$Q = 2\int_0^6 \frac{V_{max}}{b^{1/6}}\ y^{1/6}\ dy = \frac{2}{b^{1/6}}\ V_{max} \left. \frac{y^{1/6 + 6/6}}{7/6} \right]_0^b$$

$$= (2)\frac{6y^{\frac{7}{6}}}{7b^{\frac{1}{6}}}$$

$$= \frac{12}{7}\ b\ V_{max}$$

MODULE 8

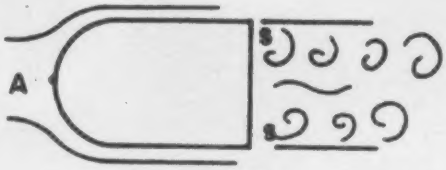
VISCOUS FLOW APPLICATIONS

SUBJECT MATTER

With the basic concepts of laminar and turbulent flow in hand, it is advantageous to consider several applications of practical importance. This will serve not only to reinforce these basic concepts but also to acquaint you with the range of "real world" problems that you can approach with these tools. Let's look briefly at these applications.

1.) Boundary Layers - When fluid flows along a boundary, the fluid close to the wall is slowed due to viscous effects. All fluid particles that are affected by the boundary comprise the so-called boundary layer. Because the velocity at the wall is zero, a velocity gradient ($\frac{du}{dy}$) exists at the wall and therefore a shear stress ($\tau = \mu \frac{du}{dy}$) exists also. Integrating the shear stress gives a shear force which is commonly called skin friction. For streamlined shapes such as airfoils, or submarine hulls this represents the predominant resistive force to motion.

2.) Flow Over Bluff Bodies - For uniform flow over a bluff body (say a road sign or a building on a windy day) the fluid on the downstream side of the body cannot follow the contour and a phenomenon called separation occurs. Note the diagram. At point A, the velocity is zero and the pressure is maximum (a stagnation point). As the fluid accelerates along the boundary contour the pressure is necessarily



reduced (why?). At point "s" the fluid cannot follow the boundary and the fluid separates from the boundary. The pressure at s is still low and thus an integration of the pressure over the surface of the body would yield a net force in the direction of flow. This force is called form drag since it depends on the shape of the body. For external flows, then, the total drag is the sum of the form drag and skin friction. For streamlined shapes, skin friction predominates, whereas for non-streamlined shapes, form drag predominates. Obviously, many problems exist where both drag components are important.

3.) Pipe and Channel Flow - Internal flows such as pipe and channel flow are of extreme importance to the engineer. Stop and consider the situations in your proposed career where a knowledge of pipe or channel flow would be of value. Perhaps you will recognize the importance of this application. The initial study in viscous flow has given you a good background in laminar pipe flow. For turbulent pipe flow, a convenient set of curves (called the Moody diagram) has been developed. A major effort will be made in the proper interpretation and use of these curves.

INCREMENTAL OBJECTIVES

At the conclusion of this module, you should be able to:

8.1) Compute the shear stress, shear force and boundary layer thickness for laminar flow over a smooth flat plate using an assumed velocity profile that satisfies the boundary conditions.

8.2) Compute the shear stress, shear force and boundary layer thickness for turbulent flow over a smooth flat plate using an assumed velocity profile that satisfies the boundary conditions.

8.3) State the definition of the drag coefficient and describe its use for flow over bluff bodies.

8.4) Use the drag coefficients shown in Figure 5.21, Figure 5.22, and Table 5.1 to compute the drag force over the Reynolds number ranges indicated in this data.

8.5) State the Darcy - Weisbach equation for head loss in pipe and open channel flow.

8.6) State the definition of the Moody friction factor for laminar and turbulent flow.

8.7) Given head loss and pipe size of a pipe flow, calculate the discharge.

8.8) Given the discharge and pipe size of a pipe flow, calculate the head loss.

8.9) Given the discharge and head loss of a pipe flow, calculate the pipe diameter.

8.10) Define and construct the energy line and hydraulic grade line for a given pipe flow problem.

PROBLEM INDEX

Problem Number	Problem Description
8.1.1:1	Laminar boundary layer
8.2.1:2	Skin friction
8.2.2:2	Boundary layer thickness
8.3.1:2 (8.4.1:2)	Terminal velocity of a sphere
8.3.2:2 (8.4.2:2)	Form drag on a parachute
8.5.1:1	Rectangular open channel flow
8.6.1:1	Discharge computation for laminar pipe flow
8.7.1:1	Discharge computation for turbulent pipe flow
8.8.1:4	Energy and power dissipation in pipe flow
8.8.2:4	Pipe roughness effect in turbulent flow
8.8.3:4	Effect of valves and fittings in turbulent pipe flow
8.8.4:4	Energy dissipation for pipes in series
8.9.1:1	Pipe size computation for turbulent pipe flow
8.10.1:1	Hydraulic and energy grade lines

PROBLEM SET #8

8.1.1:1 Using the velocity distribution $\frac{u}{U} = \sin \Pi \frac{y}{2\delta}$, determine the equation for growth of the laminar boundary layer and for shear stress along a smooth, flat plate in two-dimensional flow.

8.2.1:2 Estimate the skin-friction drag on an airship 300 feet long, average diameter 60 feet, with velocity of 80 mph traveling through air at 13 psia and 80°F.

8.2.2:2 Air at 70°F, 14.7 psia, flows along a smooth plate with a velocity of 100 mph. How long does the plate have to be to obtain a boundary-layer thickness of $\frac{1}{4}$ inch?

8.3.1:2 (8.4.1:2)

What is the terminal velocity of a 2" diameter metal ball, sp. gr. 3.5, dropped in oil, sp. gr. 0.80, $\mu = 1$ poise? What would be the terminal velocity for the same size ball but with a 7.0 sp. gr.? How do these results agree with the experiments attributed to Galileo at the Leaning Tower of Pisa?

8.3.2:2 (8.4.2:2)

How many 100' diameter parachutes ($C_D = 1.2$) should be used to drop a bulldozer weighing 10,000 lb at a terminal speed of 32 ft/sec through air at 14.5 psia, 70°F?

8.5.1:1 A rectangular channel 4 feet wide, Chézy C = 60, S = 0.0064, carries 40 cfs. Determine the velocity.

8.6.1:1 Fluid flows through a $\frac{1}{2}$" diameter tube at a Reynolds number of 1800. The head loss is 30 feet in 100 feet of tubing. Calculate the discharge in gallons per minute.

8.7.1:1 Two liquid reservoirs are connected by 200 feet of 2" diameter smooth tubing. What is the flow rate when the difference in elevation is 50 feet? $\nu = 0.001\ ft^2/sec.$

8.8.1:4 Water at 70°F is to be pumped through 1200 feet of 8" diameter wrought-iron pipe at the rate of 800 gpm. Compute the head loss and horsepower required.

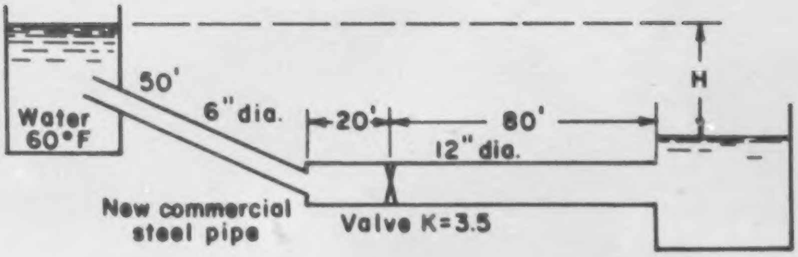
8.8.2:4 An old pipe 72" in diameter has a roughness of $\varepsilon = 0.1$. A $\frac{1}{2}$" thick lining would reduce the roughness to $\varepsilon = 0.0004$. How much in pumping costs would be saved per year per 1000 feet of pipe for water at 70°F with velocity of 8 ft/sec? The pumps and motors are 80 per cent efficient and power costs 1 cent per kilowatthour.

8.8.3:4 A water line connecting two reservoirs at 70°F has 5000 feet of 24" diameter steel pipe, three standard elbows, a globe valve, and a re-entrant pipe entrance. What is the difference in reservoir elevations for 20 cfs?

8.8.4:4 Two reservoirs are connected by three commercial steel pipes in series, $L_1 = 1000$ ft, $D_1 = 8$ in; $L_2 = 1200$ ft, $D_2 = 1$ ft; $L_3 = 4000$ ft, $D_3 = 18$ in. When $Q = 3$ cfs water at 70°F, determine the difference in elevation of the reservoirs.

8.9.1:1 In a process 10,000 lb/hr of distilled water at 70°F is conducted through a smooth tube between two reservoirs having a distance between them of 30 feet and a difference in elevation of 4 feet. What size tubing is needed?

8.10.1:1 Sketch the hydraulic and energy grade lines for the figure below, H = 24 feet.


50'
6" dia.
Water
60°F
20'
80'
12" dia.
H
New commercial
steel pipe
Valve K=3.5

PROBLEM SET #8--ANSWERS

8.1.1:1 $\frac{u}{U} = F\left(\frac{y}{\delta}\right) = \sin\frac{\Pi}{2}\eta$

laminar boundary layer

$$\tau_o = \rho V^2 \frac{d\delta}{dx}\int_o^1 F(1 - F)\,d\eta$$

$$\tau_o = \rho V^2 \frac{d\delta}{dx}\int_o^1 \sin\frac{\pi}{2}\eta\left(1 - \sin\frac{\Pi}{2}\eta\right) d\eta$$

$$\tau_o = \rho V^2 \frac{d\delta}{dx}\frac{2}{\Pi}\int_o^1 \left(\sin\frac{\Pi}{2}\eta - \sin^2\frac{\Pi}{2}\eta\right) d\eta\,\frac{\Pi}{2}$$

$$= \rho V^2 \frac{d\delta}{dx}\frac{2}{\Pi}\left[-\cos\frac{\Pi}{2}\eta\right]_o^1 + \rho V^2 \frac{d\delta}{dx}\int_o^1 \left(-\frac{1}{2} + \frac{\cos \Pi\eta}{2}\right) d\eta$$

$$+ \rho V^2 \frac{d\delta}{dx}\left(-\frac{\eta}{2})\right]_o^1 + \rho V^2 \frac{d\delta}{dx}\left(+\frac{\sin \Pi\eta}{2})\right]_o^1$$

but, $\left.+\frac{\sin \Pi\eta}{2}\right]_o^1 = 0$

therefore,

$$\tau_o = \rho V^2 \frac{2}{\Pi}\frac{d\delta}{dx}(0 + 1) + \rho V^2 \frac{d\delta}{dx}\left(-\frac{1}{2}\right)$$

$$\tau_o = \rho V^2 \frac{d\delta}{dx}\left(\frac{2}{\Pi} - \frac{1}{2}\right) = \rho V^2 \frac{d\delta}{dx}(.1366)$$

also, $\tau_o + \mu\frac{V}{\delta}\frac{\partial u_{/U}}{\partial y_{/\delta}} = \mu\frac{V}{\delta}\frac{\rho(u_{/U})}{\partial(y_{/\delta})}$

$$= \mu\frac{V}{\delta}\left.\cos\frac{\Pi}{2}\eta\,\frac{\Pi}{2}\right]_{y = 0}$$

$$= \mu\frac{V}{\delta}\frac{\Pi}{2}$$

now, $\rho V^2 \frac{d\delta}{dx}(.1366) = \mu\frac{V}{\delta}\frac{\Pi}{2}$

$$\frac{d\delta}{dx} = \mu \frac{\Pi}{2} \frac{1}{\rho V(.1366)}$$

$$\delta d\delta \frac{[\rho V^2(.1366)]}{\mu V \frac{\Pi}{2}} = dx$$

$$\frac{\delta^2}{2} [\frac{\rho V}{\mu} (.0869)] = x + C \qquad C = 0$$

$$\delta = \sqrt{\frac{\mu}{\rho V} x\ (22.99)} \quad = \frac{4.795}{Re_x^{1/2}} \cdot x$$

Then

$$\tau_o = \mu \frac{V}{\delta} \frac{\Pi}{2}$$

$$= \mu \frac{V}{4.795} (\frac{\rho V}{\mu x})^{1/2} \frac{\Pi}{2}$$

$$= .327 \frac{\mu^{1/2} V^{3/2} \rho^{1/2}}{x^{1/2}}$$

$$= .327 \sqrt{\frac{\mu V^3 \rho}{x}}$$

8.2.1:2 $L = 300$ feet long

$U = 80$ mph x $\frac{44}{30} = 117.3$ fps

$P = 13$ psia , $t = 80^{0}F$ or $80 + 460 = 540^{o}R$

$$\rho = \frac{13(144)}{53.3(540)} = .0650 = .00202 \text{ slug/ft}^3$$

$$\mu = 4 \times 10^{-7} \ \frac{\text{lb-sec}}{\text{ft}^2}$$

$$\nu = \frac{4 \times 10^{-7}}{2.01 \times 10^{-3}} 1.98 \times 10^{-4} \ \frac{\text{ft}^2}{\text{sec}}$$

$$Re = \frac{VD}{\nu} = \frac{117.3(L)}{1.99} \times 10^4$$

$$= \frac{117.3(300)}{1.99}$$

$$= 1.777 \times 10^8$$

at 3 ft, $Re = 1.777 \times 10^6$

Boundary layer is completely turbulent

$$C_D = \frac{.455}{(\log_{10} Re)2.58} = 1.96 \times 10^{-3} \quad \text{where } Re = 1.777 \times 10^8$$

$$\log_{10}(1.768)10^8 = 8 + \log_{10}(1.768)$$

$$= 8.24748$$

$$\text{Force} = C_D \text{Area} \ \rho\frac{V^2}{2}$$

$$= C_D(60\pi)\ 300(.00201)\frac{(117.3)^2}{2}$$

1532.6 lb

8.2.2:2 air at 70°F, 14.7 psia

$\nu = 1.75 \times 10^{-4}$ ft^2/sec

$$U = 100(\frac{44}{30}) = \frac{10}{3}(44) = \frac{440}{3} = 147 \text{ fps}$$

assume laminar boundary layer

$$Re = 500{,}000 = \frac{UL}{\nu}, \quad L = \frac{Re(\nu)}{U}$$

$$= \frac{500{,}000 \times 1.75 \times 10^{-4}}{147}$$

$$= .595 \text{ ft}$$

$$\frac{\delta^2}{2} = 10.78 \frac{\nu}{U} x$$

at L = .595 ft

$$\delta = [2(10.78) \frac{\nu}{U} x]^{1/2}$$

substituting, $\delta = .0468$ inches

Therefore the boundary layer must be turbulent

for turbulent boundary layer

$$\delta = .37 (\frac{\nu}{U})^{1/5} x^{4/5}$$

$$(\frac{1}{48}) = .37 (\frac{1.75 \times 10^{-4}}{147})^{1/5} x^{4/5}$$

$$10 [\frac{14.70 \times 10^5}{1.75}]^{1/5} \frac{1}{48(.37)} = x^{4/5}$$

.830 ft ∿ 9.96 inches

8.3.1:2 (8.4.1:2)

metal ball sp. gr. = 3.50

oil sp. gr. = 0.80

μ = 1 poise

Stokes law

$$U = \frac{2}{g} \frac{a^2}{\mu} (\gamma_{ball} - \gamma_{oil})$$

$$U = \frac{2}{g} \frac{(\frac{1}{12})^2}{1} (479)(3.5 - .8)(62.4)$$

$$U = \frac{2}{9(144)} (479)(2.7)(62.4)$$

U = 124.5 ft/sec

$$Re = \frac{VD}{\nu}$$

$$= \frac{498.16\ (1)}{6\ (1)}\ (1.94)(.8)\ 479$$

turbulent, therefore cannot use Stokes law

W - Drag - Buoyant force = 0

$$\gamma_S V - \gamma_o V - C_D A \frac{\rho V^2}{2} = 0$$

$(\gamma_S - \gamma_o)V = C_D A \frac{\rho V^2}{2}$ but C_D depends on V through curve Re vs C_D

therefore assume a Re = 1; C_D, V are determined

$U = 7.6$ fps $C_D = .417$ $Re = 941$

Solve until both equations are satisfied.

8.3.2:2 (8.4.2:2)

Drag coefficient, $C_D = 1.2$

Pressure = 14.5 psia

Temperature = 70°F , 530°R

Velocity = 32 fps

Weight of bulldozer = 10,000 lb

Density of air can be found by Ideal gas equation.

$$\rho = \frac{P}{RT}$$

$$\rho = \frac{14.5 \times 144}{53.3 \times 530} = 0.07391 \text{ lb/ft}^3$$

Drag = $C_D \rho \frac{V^2}{2} \times A_n$ where A_n is projected area

$$= \frac{1.2 \times .07391 \frac{lb_m}{ft^3}}{32.2 \frac{lb_m}{lb_F} \frac{ft}{sec^2}} \times \frac{(32)^2 \frac{ft^2}{sec^2}}{2} \times (\Pi \times 50 \times 50)\ ft^2$$

= 11,071 lb_F which is greater than 10,000 lb_F

therefore, the number of balloons = 1.

8.5.1:1 rectangular channel

Chezy C = 60

$S = 0.0064$

$Q = 40$ cfs

What is velocity?

equation 5.8.3 , page 277 of Reference #1, $V = C\sqrt{RS}$

where R = hydraulic radius = $\frac{A}{P} = \frac{4y}{4 + 2y}$

Multiply both sides of equation by A, area

$$VA = C\sqrt{RS} \times A$$

$$Q = C\sqrt{RS} \times A$$

$$40 = 60\sqrt{\frac{4y}{4 + 2y}(.0064)}\ (4y)$$

$$40 = 60\,(4y)(.0064)^{1/2}\left(\frac{4y}{4 + 2y}\right)^{1/2}$$

simplifying

$$8.375 = (4y)\left(\frac{4y}{4 + 2y}\right)^{1/2}$$

trial & error, y = 2.07 feet, height of water.

To determine the velocity, $Q = VA$

therefore, $V = \frac{Q}{A}$

$= \frac{40}{4\ (2.07)}$, V = 4.83 ft/sec

8.6.1:1 $D = \frac{1}{2}$ inch

$Re = 1800$

$h_L = 30 \frac{\text{ft-lb}}{\text{lb}}$ in 100 feet

What is the discharge?

Since flow is laminar, $f = \frac{64}{Re}$

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

$$30 = \frac{64}{1800} (100)(24) \frac{V^2}{64.4}$$

$$V^2 = \frac{30(1800)(64.4)}{2400(64)}$$

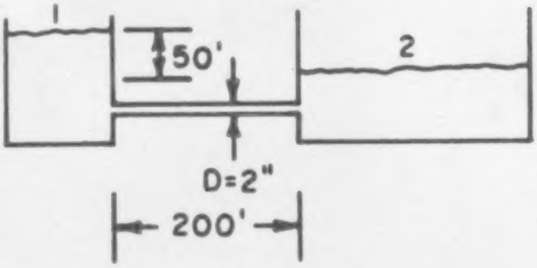
$$V^2 = 22.64 \quad , \qquad V = 4.758$$

$$Q = VA = 4.758 \frac{\pi}{4} \left[\frac{1}{24}\right]^2$$

$$= 4.758 \frac{\pi}{4} \left(\frac{1}{24}\right)\left(\frac{1}{24}\right) = 6.49 \times 10^{-3} \text{ cfs}$$

$$= 6.49 \times 10^{-3} \times 7.48 \frac{\text{gal}}{\text{ft}^3} \times 60 \frac{\text{sec}}{\text{min}}$$

$$= 2.91 \text{ gpm}$$



8.7.1:1 L = 200 feet, D = 2 inches

elevation difference = 50 feet

$\nu = 0.001$ ft^2/sec

Smooth tubing , $\varepsilon = 0$ H = 50 feet

$$\frac{P_1}{\gamma} \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + L_{1-2}$$

$$L_{1-2} = 50$$

losses = Σ (entrance losses + pipe friction + exit) = 50 feet

$$50 = 0 + (f \frac{L}{D} \frac{V^2}{2g}) + 1 \frac{V^2}{2g}$$

$$50 = (f \frac{L}{D} + 1) \frac{V^2}{2g}$$

Trial Solution

1.) $f = .018$, for $Re = 10^5$

$$50 = [1200 (.018) + 1] \frac{V^2}{2g}$$

$$50 = \frac{22.6}{64.4} V^2$$

$$V = \sqrt{\frac{50 (64.4)}{22.6}} = 11.93 \text{ fps}$$

$$Re = \frac{11.93 (2)}{12 (.001)} = 1988 \quad \text{, laminar}$$

2.) $Re = 10^4$, $f = .031$

$$50 = [1200 (.031) + 1] \frac{V^2}{2g} = \frac{38.2}{64.4} V^2$$

$$V = \sqrt{50 (\frac{64.4}{38.2})} = 9.18 \text{ fps}$$

$$Re = \frac{9.18 (2)}{12(.001)} = 1530 \quad \text{try again since } 1530 << 10^4$$

keep selecting values of Re, then obtain the value of f from Reference #1. When calculated Re ∿ selected Re, value of V is correct for Re = 1175 , V = 7.07 fps

$Q = .0218 \, \alpha \, V = .154 \text{ ft}^3/\text{sec}$

8.8.1:4 Water @ 70°F

$L = 1200$ feet

$D = 8$ inches (wrought iron pipe)

rate = 800 gpm

compute head loss and HP required

$$800 \text{ gpm } \left(\frac{1}{(7.48)(60)}\right) = 1.78 \text{ cfs}$$

$$V = \frac{1.78}{.349} = 5.11 \text{ fps}$$

$$h_L = f\,\frac{L}{D}\,\frac{V^2}{2g} \quad \text{where } \frac{\varepsilon}{D} = \frac{.00015}{\frac{8}{12}} = 2.25 \times 10^{-4}$$

$$Re = \frac{VD}{\nu} = \frac{\frac{2}{3}(5.11)}{1.059} \times 10^5 = 3.22 \times 10^5$$

$$f = .0165$$

$$h_L = .0165 \;\; 1200\,(1.5)\,\frac{(5.11)^2}{64.4}$$

$$h_L = 12.0\,\frac{\text{ft-lb}}{\text{lb}_m}$$

$$\text{Hydraulic HP} = \frac{h_L W}{550} = \frac{12.0(1.78)(62.4)}{550}$$

$$= 2.43 \text{ hp}$$

8.8.2:4

old pipe	new pipe
72" in diameter	71" in diameter
$\varepsilon = .1$	$\varepsilon = .0004$

Water @ 70°F, $\nu = 1.059 \times 10^{-5}$ ft^2/sec

$\gamma = 62.3$ lb/ft^3

L = 1000 feet

V = 8 ft/sec

for old pipe

$$Re = \frac{VD}{\nu} = \frac{8(6) \times 10^5}{1.059}$$

$$Re = 4.53 \times 10^6$$

$$\frac{\varepsilon}{D} = \frac{.1}{6} = .0167$$

$$f = .046$$

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

$$= .046\,\frac{(1000)}{6}\,\frac{(64)}{(64.4)}$$

$$h_f = \frac{46}{6}\left(\frac{64}{64.4}\right) = 7.62\ \frac{\text{ft-lb}}{\text{lb}}$$

$$\text{power} = \frac{(\gamma AV)h_f}{550} = \frac{226(62.3)\ 7.62}{550}$$

power actual = 243.8 hp

energy = 243.8 x 1 hr = 243.8 hp - hr

$$Q = \Pi\,(3)^2\,\gamma$$

$$= 226\ \text{ft}^3/\text{sec}$$

For 1¢/kw-hr:

energy = 181.8 kw-hr x $.01/kw-hr = 1.818

cost/yr = 1.818 x 24 x 365 = $15,945

for new pipe

$$Re = \frac{8\left(\frac{71}{12}\right)}{1.059} = 4.47 \times 10^6$$

$$\frac{\varepsilon}{D} = \frac{.0004}{\frac{71}{12}} = .0000676$$

$$f = .0117$$

$$h_f = .0117\left(\frac{1000}{\frac{71}{12}}\right)\frac{64}{64.4}$$

$$= \frac{11.7}{\frac{71}{12}} \times \frac{64}{64.4} = 1.97\ \frac{\text{ft-lb}}{\text{lb}}$$

power = $(\gamma AV)\ 1.97$

$$\text{power(actual)} = \frac{219.95(62.3)(1.97)}{550(.8)} = \frac{49.08}{.8}\ \text{hp}$$

energy = 61.35 x 1 hr = 61.35 hp-hr

Q = 219.95 ft^3/sec , 1 hp = .746 kw

$$61.35\ \text{hp-hr} \times \frac{.746\ \text{kw}}{\text{hp}} = 47.56\ \text{kw hr}$$

Also 45.76 x .01 = .4576

cost/yr = .4576 x 24 x 365 = $4009

savings = 15,945 - 4009 = $11,936

8.8.3:4 Water @ 70°F

$\nu = 1.059 \times 10^{-5}$ ft^2/sec

$\rho = 1.935$

5000 feet of 24" diameter steel pipe, $\varepsilon = .00015$

3 standard elbows $\frac{\varepsilon}{D} = .000075$

1 globe valve

1 re-entrant pipe entrance

$H = \Sigma$ losses

$$H = f\,\frac{5000}{2}\,\frac{V^2}{2g} + \overset{\text{entrance}}{1\,\frac{V^2}{2g}} + \overset{\text{exit}}{\frac{V^2}{2g}} + \overset{\text{valve}}{10\frac{V^2}{2g}} + \overset{\text{elbow}}{3(.9)\frac{V^2}{2g}}$$

$$= \frac{V^2}{2g}\,[f(2500) + 2 + 10 + 2.7]$$

$Q = 20 = VA$

$$V = \frac{20}{A} = \frac{20}{3.14} = 6.37 \text{ fps}$$

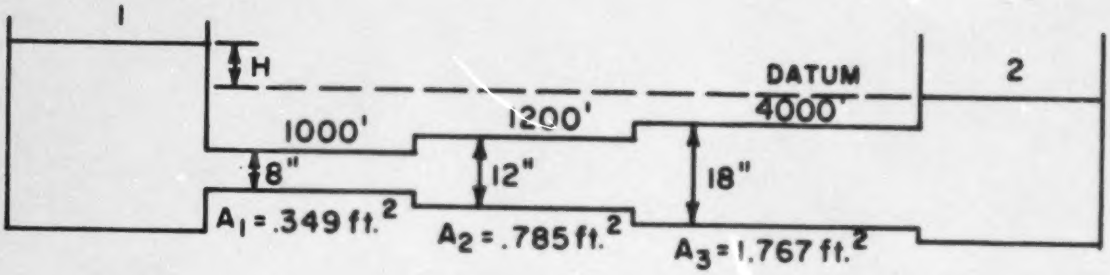
$$Re = \frac{VD}{\nu} = \frac{6.37(2)(10^5)}{1.059} = 12 \times 10^5 = 1.2 \times 10^6$$

$f = .0128$

$$H = [.0128\,(2500) + 1 + 1 + 10 + 2.7]\,\frac{(6.37)^2}{64.4}$$

$$H = 29.42\,\frac{\text{ft-lb}}{\text{lb}}$$

8.8.4:4


1
H
DATUM
2
1000'
1200'
4000'
8"
12"
18"
A1 = .349 ft.2
A2 = .785 ft.2
A3 = 1.767 ft.2

@ 70°F $, \nu = 1.059 \times 10^{-5}$ ft^2/sec

$\rho = 1.936$ slugs/ft^3

$\gamma = 62.3$ lb/ft^3

$V_1 = 8.6$ fps

$V_2 = \dfrac{3}{\left(\dfrac{\Pi\,(1)^2}{4}\right)} = 3.82$ fps $\qquad V_3 = \dfrac{3}{\left(\dfrac{\Pi\,(1.5)^2}{4}\right)} = 1.698$ fps

For pipe D = 8"

$\dfrac{\varepsilon_1}{D_1} = .000225 \qquad Re_1 = \dfrac{8.6(.667)(10^5)}{1.059} = 5.41 \times 10^5$

$f = .0155$

For pipe D = 12"

$\dfrac{\varepsilon}{D_2} = \dfrac{.00015}{(\frac{12}{12})} = .00015 \qquad Re_2 = 3.61 \times 10^5$

$f_2 = .0155$

For pipe D = 18"

$\dfrac{\varepsilon}{D_3} = \dfrac{.00015}{(\frac{18}{12})} = .0001 \qquad Re = \dfrac{1.698(1.5)}{1.059} \times 10^5 = 2.41 \times 10^5$

f = .016[illegible]

entrance losses = $0.5(\frac{V_1^2}{2g})$

sudden expansion $h_L = K \frac{V_1^2}{2g}$ (note 1 is upstream velocity)

$$K = [1 - (\frac{D_1}{D_2})^2]^2$$

energy equation 1-2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + H = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \Sigma h_{L_{1-2}}$$

$$H = \Sigma h_{1-2} = h_{entrance} + h_{pipe\ 1} + h_{exp\ 1-2} + h_{pipe\ 2} + h_{exp\ 2-3} + h_{pipe3} + h_{exit}$$

$$H = (.5)\frac{V_1^2}{2g} + (.0155)\frac{1000}{.667}\frac{V_1^2}{2g} + (.308)\frac{V_1^2}{2g} + (.0155)(\frac{1200}{1})\frac{V_2^2}{2g} + (.308)\frac{V_2^2}{2g} + (.01627)(\frac{4000}{1.5}) + \frac{V_3^2}{2g} + (1)\frac{V_3^2}{2g}$$

$$H = \frac{V_1^2}{2g}[.5 + 23.2 + .308] + \frac{V_2^2}{2g}[18.91] + \frac{V_3^2}{2g}[43.2 + 1]$$

$$H = 27.61 + 4.28 + 1.97 = 32.86 \frac{ft\text{-}lb}{lb} \quad \text{or feet}$$

$H = 33.87$ ft

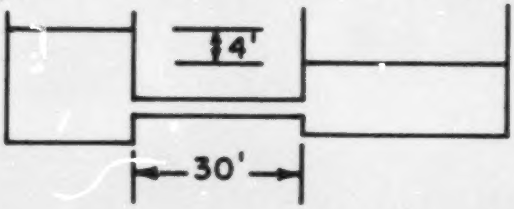
8.9.1:1 W = 10,000 lb/hr

Water @ 70°F , $\nu = 1.059 \times 10^{-5}$ ft^2/sec

$\rho = 1.94$ slugs/ft^3

L = 30 feet

elevation difference = 4 feet



$$Q = 10{,}000 \text{ lb/hr}$$

$$= \frac{10{,}000 \text{ lb}}{3600 \text{ sec}} \times \frac{1}{62.4} = 4.45 \times 10^{-3} \text{ cfs}$$

$$Q = .0463 \text{ cfs}$$

$$V = \frac{Q}{A} = \frac{576}{\pi} \frac{(.0463)}{D^2}$$

$$V = \frac{8.49}{D^2}$$

smooth tube $\varepsilon = 0$

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

$$4 = f \frac{L}{D} \frac{V^2}{2g}$$

$$4 = f \frac{30}{D^5} \frac{(.00463)^2}{644}$$

$$D^5 = C_1(f) = f\left(\frac{30}{4}\right) \frac{(.00463)^2}{64.4}$$

$$D^5 = 2.5 \times 10^{-4}(f)$$

$Re = \frac{VD}{\nu}$, substituting the expressions for V, & D

$$Re = \frac{.0463}{D^2} \cdot \frac{D}{1.059} 10^5$$

$$= \frac{4630}{1.059\ D} = \frac{4370}{D}$$

Assume a value of f, then obtain value for Re. Plug into equation above to obtain D.

Correct solution

$$f = .0205$$

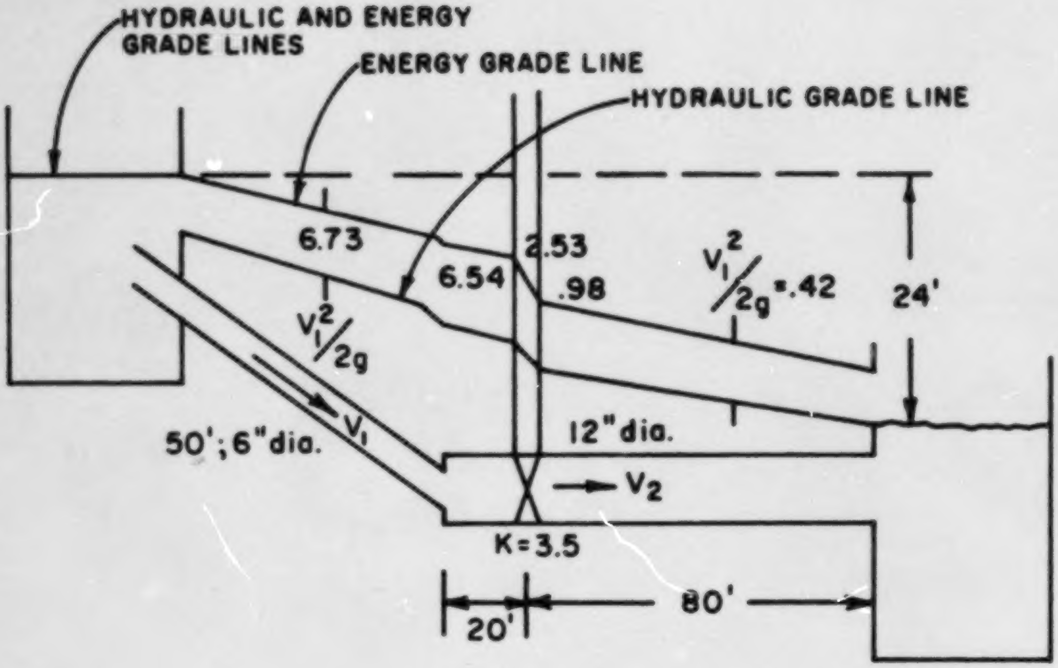
$$Re = 5.09 \times 10^4$$

$$D = \frac{4370}{5.09 \times 10^4}$$

$$D = .085 \text{ ft}$$

$$D = 1.03 \text{ in}$$

8.10.1:1


HYDRAULIC AND ENERGY GRADE LINES
ENERGY GRADE LINE
HYDRAULIC GRADE LINE
6.73
6.54
2.53
.98
$V_1^2/2g$
$V_1^2/2g = .42$
24'
V_1
50'; 6" dia.
12" dia.
V_2
K=3.5
20'
80'

RESOURCE MATERIAL

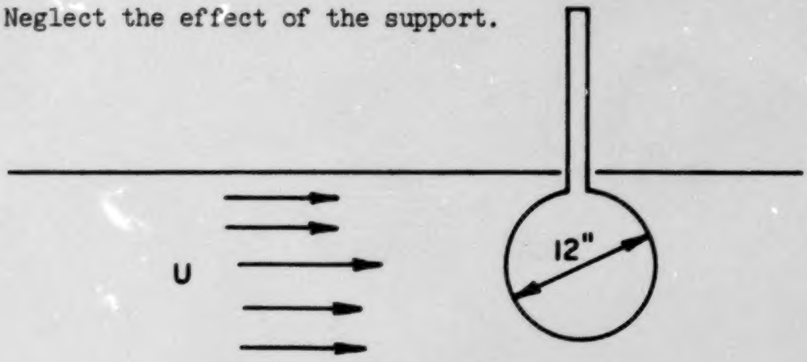
Ref. No.	Chapter	Pages	Other Description
1	5	258-299	Articles 5.6-5.10
9			FM-33 Film Loop
10			FM-4 Film Loop
11			FM-4 Film Loop
12			Technical paper-"Valves, Fittings, and Pipes"

QUIZ

1.) Given the velocity profile $\frac{u}{U} = \eta - 3\eta^2 + 2\eta^3$

for a laminar boundary layer, where $\eta = \frac{y}{\delta}$. Would you expect this profile to give the same skin friction drag (i.e., good agreement with exact results) as those that you have studied previously? Explain

2.) A hand held meter for determining the velocity in an open channel consists of a sphere on a support as shown. Determine the flow velocity if the force on the sphere is 7.3 lb when the water temperature is 70°F. Neglect the effect of the support.



3.) Water flows through a riveted steel pipe of 5" ID at an average velocity of 12 fps. Water temperature is 50°F. For these conditions determine:

a.) friction factor

b.) wall shear stress

c.) pressure drop in 50 feet of pipe

QUIZ ANSWERS

1.) $\frac{u}{U} = \eta - 3\eta^2 + 2\eta^3$

Boundary condition #1; $y = 0$, $u = 0$ O.K.

Boundary condition #2; $y = 1$, $u = 0$ no good!

The profile will not work.

2.) $\rho = 1.936$ slugs/ft^3 , $\nu = 1.059 \times 10^{-5}$ ft^2/sec

$$F = C_D A_p \frac{1}{2} \rho V^2$$

where $C_D = f\,(\mathrm{Re})$ from Figure 5.21 of Reference #1

$$\mathrm{Re} = 4 \times 10^5$$

$$V = \frac{\mathrm{Re}\,V}{D} = \frac{4 \times 10^5\,(1.059)\,10^{-5}}{1}$$

$$V = 4.236 \text{ ft/sec}$$

$$C_D = .2$$

$$F = .2\left(\frac{\Pi}{4}\right)\left(\frac{1}{2}\right)(1.936)(4.236)^2$$

$F = 2.73$ lb

Assume $\mathrm{Re} = 6 \times 10^5$, $V = 6.354$ ft/sec

$$F = .2\left(\frac{\Pi}{4}\right)\left(\frac{1}{2}\right)(1.936)(6.37)^2 = 6.13 \text{ lb}$$

Assume $\mathrm{Re} = 6.5 \times 10^5$, $V = 6.88$

$$F = .2\left(\frac{\Pi}{4}\right)\left(\frac{1}{2}\right)(1.936)(6.88)^2 = 7.20 \text{ lb} \quad \underline{\text{Close}}$$

$V \simeq 6.88$ ft/sec

3 a.) $\mathrm{Re} = \frac{VL}{\nu}$

$$= \frac{(12)(\frac{5}{12})}{1.41 \times 10^{-5}} = 3.55 \times 10^{5}$$

ε for steel = .0165 ft

$$\frac{\varepsilon}{D} = \frac{.0165 \text{ ft}}{\frac{5}{12} \text{ ft}} = .0396$$

f (from chart) = 0.064 Figure 5.32 (Moody diagram), Reference #1

b.) $\sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{f}{8}}\,(V)$

$$\sqrt{\frac{\tau_o}{1.935}} = \sqrt{\frac{.064}{8}}\,(12)$$

$$\tau_o = 2.23 \text{ lb/ft}^2$$

c.) $\frac{\Delta P}{\gamma} = f\,\frac{L}{D}\,\frac{V^2}{2g}$

$$\frac{\Delta P}{62.4} = (0.064)\left(\frac{50}{\frac{5}{12}}\right)\left(\frac{(12)^2}{(2)(32.2)}\right)$$

$$\Delta P = 1071 \text{ lb/ft}^2$$

MODULE 9

COMPRESSIBLE FLUID FLOW

SUBJECT MATTER

Throughout most of the discussions in fluid dynamics you have assumed the fluid to be incompressible. You did look at the perfect gas law rather briefly in Module 1. This equation expresses the relationship between pressure, density and temperature.

In this module, you shall consider fluid compressibility. In other words, as a fluid particle moves along its path in a flow, its density or mass/volume can change depending on the local temperature and pressure. Since temperature changes occur heat can be transferred and the internal energy will vary. For a perfect gas, recall that internal energy is a function of temperature only. Therefore, an added complication results when the compressibility of a fluid (usually a gas) becomes important since the basic equations must be altered. For example:

1) a. Continuity equations for incompressible steady pipe flow

$$Q = A_1V_1 = A_2V_2$$

b. Continuity equations for compressible steady pipe flow

$$M = \rho_1A_1V_1 = \rho_2A_2V_2$$

2) a. Energy equations for steady incompressible pipe flow

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{1-2}$$

b. Energy equation for steady compressible pipe flow

$$\int \frac{dp}{\rho} + V_1^2 + gZ_1 = \int \frac{dp}{\rho} + V_2^2 + gZ_2 + h_{1-2}$$

Note that before a final end point equation can be determined, the variation of ρ with the pressure must be known. Although there are many possible relationships which adequately model compressible flows of engineering interest, you will investigate only the isentropic process. The application that you shall consider is the flow of a gas from a reservoir through a converging nozzle. If the tank pressure is large enough and the flow cross section small enough (at the nozzle throat) you can obtain sonic flow (Mach number = 1). Thrusters for jet aircraft and spacecrafts are examples of hardware utilizing this principle.

INCREMENTAL OBJECTIVES

At the conclusion of this module, you should be able to:

9.1) From recall, state the basic equations for internal energy, enthalpy, entropy, sonic velocity, and the equation of state for a perfect gas.

9.2) State the definition of a polytropic process and list the defining equations for the following:

- a) constant temperature process
- b) constant pressure process
- c) constant volume process
- d) constant entropy process

9.3) From recall, state the continuity equation and the two forms of the energy equation which apply to isentropic flow of a perfect gas through a converging nozzle.

9.4) Given the fluid and its temperature and pressure in the reservoir, compute the pressure, temperature, density and velocity anywhere in the converging nozzle when the back pressure is specified.

PROBLEM INDEX

Problem Number	Problem Description
9.1.1:4	Specific heat computation
9.1.2:4	Enthalpy computation of an isentropic process in a perfect gas
9.1.3:4	Mach number computation
9.1.4:4	Sonic velocity of a perfect gas
9.2.1:1	Pressure computation of an isentropic process in a perfect gas
9.3.1:2 (9.4.1:2)	Isentropic flow through a converging nozzle
9.3.2:2 (9.4.2:2)	Isentropic flow through a converging nozzle

PROBLEM SET #9

9.1.1:4 Three pounds mass of a perfect gas, molecular weight 36, had its temperature increased 3.2°F when 2,000 ft-lb work was done on it in an insulated constant-volume chamber. Determine c_v and c_p.

9.1.2:4 Compute the enthalpy change per slug for helium from T_1 = 0°F, P_1 = 15 psia to T_2 = 100°F in an insentropic process.

9.1.3:4 A projectile moves through water at 80°F at 3,000 ft/sec. What is its Mach number?

9.1.4:4 What is the speed of sound through hydrogen at 80°F?

9.2.1:1 In an isentropic process 3 lb_m oxygen with a volume of 4.5 ft^3 at 60°F has its absolute pressure doubled. What is the final temperature?

9.3.1:2 (9.4.1:2)

Oxygen flows from a reservoir where P_o = 100 psia, T_o = 60°F, to a 6-in.-diameter section where the velocity is 600 ft/sec. Calculate the mass rate of flow (isentropic) and the Mach number, pressure, and temperature at the 6-inch section.

9.3.2:2 (9.4.2:2)

Air in a reservoir at 350 psia, T = 290°F, flows through a 2-in.-diameter throat in a converging-diverging nozzle. For M = 1 at the throat, calculate P, ρ, and T there.

PROBLEM SET #9--ANSWERS

9.1.1:4 a) 3 lb_m = mass

$\Delta T = 3.2$

molecular weight = 36

1st law

$Q - W = (E_2 - E_1)$ where $Q = 0$ $E = U = mc_v \Delta$

$+ W = mc_v \Delta T$

$2000 = 3c_v(3.2)$

$c_v = \frac{2000}{9.6}$

$c_v = \frac{2000}{9.6(778)} = .268 \frac{\text{Btu}}{\text{lb}_m{}^\circ\text{R}}$

b) $R = \frac{1545}{36} = 42.9$

$\frac{R}{778} = c_p - c_v$

$.055 = c_p - .268$

$c_p = .323 \frac{\text{Btu}}{\text{lb}_m{}^\circ\text{R}}$

9.1.2:4 enthalpy change $\Delta h = c_p \Delta T$

$\Delta h = 1.25 \, [100 - 0]$

$\Delta h = 125 \frac{\text{Btu}}{\text{lb}_m{}^\circ\text{R}}$

$\Delta h = 125 \frac{\text{Btu}}{\text{lb}_m{}^\circ\text{R}} \times 32.2 \frac{\text{lb}_m}{\text{slug}} = 4025 \frac{\text{Btu}}{\text{slug }^\circ\text{R}}$

9.1.3:4 Projectile moves through water at 80°F

velocity = 3,000 ft/sec

compute the Mach number

$$M = \frac{V}{C}$$

$$C = \sqrt{\frac{K}{\rho}}$$

from table C.1 of Reference number 1 at 80° $K = 322 \times 10^3$ lb/in^2

$$C = \sqrt{\frac{30(144)}{1.934}} \times 10^2$$

$$C = 4729$$

therefore

$$M = \frac{3000}{4729} = .634$$

9.1.4:4 Speed of sound of hydrogen at 80°F?

$$C = \sqrt{KRgT} = \sqrt{1.4(766)(32.2)(540)}$$

$C = 4318$ Fps

9.2.1:1 3 lb$_m$, O_2

$V = 4.5$ ft^3 at 60 F

$R = 48.3$

$K = 1.40$

$T_1 = 460 + 60 = 520°R$

$$\rho = \frac{\text{mass}}{V} = \frac{3}{4.5} \frac{\text{lb}_m}{\text{ft}^3} \quad \text{at } 60°F$$

$$P = \rho RT = \frac{3}{4.5}(48.3)(520)$$

$$P_2 = 2P_1$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}} = (2)^{\frac{.4}{1.4}} = (2)^{\frac{1}{3.5}}$$

$$T_2 = 520\ (2)^{\frac{1}{3.5}} = 633.9°R$$

$$T_2 = 174°F$$

9.3.1:1 (9.4.1:2)

oxygen

$P_o = 100$ psia

6 inch section where

$V = 600$ ft/sec

$T_o = 60°F = 520°R$

$R = 48.3 \frac{ft\ lb}{lb_m\ °R}$

$c_p = .219$

$K = 1.4$

energy equation

$$h_o = h_1 + \frac{V_1^2}{2g}$$

$$T_o = T_1 + \frac{V_1^2}{2gVCp}$$

$$520 = T_1 + \frac{(600)^2}{2(32.2)(778)(.219)}$$

$$520 = T_1 + 32.8$$

$$T_1 = 487.2°R$$

$$T_1 = 27.2°F$$

$$\frac{P_1}{P_o} = \left(\frac{T_1}{T_o}\right)^{\frac{K}{K-1}}$$

$$P_1 = 100\left(\frac{487.2}{520}\right)^{\frac{1.4}{.4}} = 79.6 \text{ psia}$$

$$\frac{P_1}{P_o} = \frac{79.6}{100} = .796$$

where as $\frac{P_*}{P_o} = .528$

therefore flow is not choked or 6" section is not minimum section

$$\rho_1 = \frac{P}{RT} = \frac{79.6(144)}{48.3(487.2)} = .487 \ \frac{\text{lb}_m}{\text{ft}^3}$$

mass flow = $(.196)(600)(.498) = 57.2 \ \frac{\text{lb}_m}{\text{sec}}$ or $1.779 \ \frac{\text{slug}}{\text{sec}}$

$$M_1 = \frac{V}{C}$$

where $C = \sqrt{KRTg} = \sqrt{(1.4)(48.3)(487.2)(32.2)} = 1030$

$$M_1 = \frac{600}{1030} = .583$$

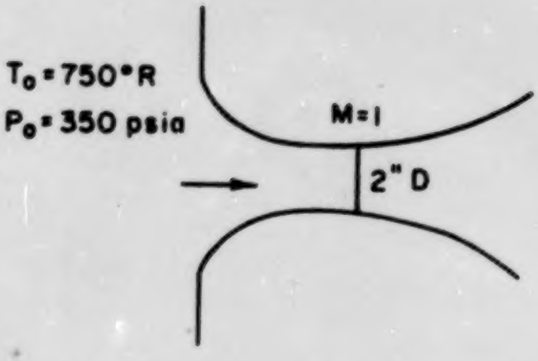
9.3.2:2 (9.4.2:2)

Air in a reservoir at 350 psia

T = 290°F or 750°R

T_o = 750°R

P_o = 350 psia



if M = 1, at throat, compute T, ρ, P

a) $\frac{T_*}{T_o} = 0.833$

$T_* - 0.833\,(750) = 624.75°R$ or $164.75°F$

b) $\frac{P_*}{P_o} = 0.528$

$P_* = 0.528\,(350) = 184.8$ psia

c) $\frac{\rho_*}{\rho_o} = .634$

$$\rho_* = .799\ \frac{lb_m}{ft^3}$$

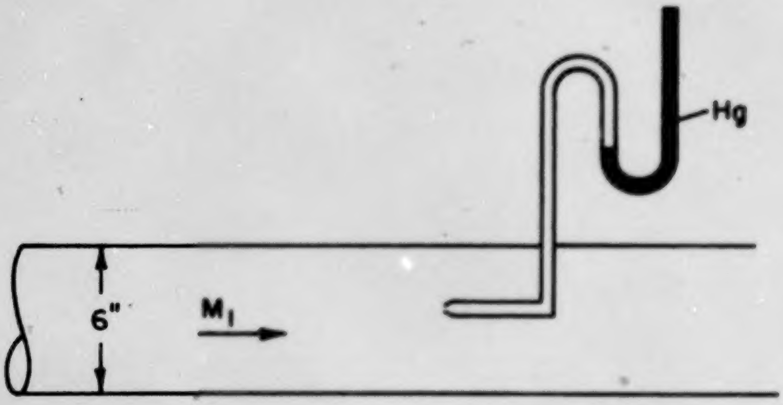
$$\rho_o = \frac{P_{0(144)}}{RT_o} = \frac{350(144)}{53.3(750)} = 1.26\ \frac{lb_m}{ft^3}$$

RESOURCE MATERIAL

Ref. No.	Chapter	Pages	Other Description
1	6	325-341	Articles 6.1-6.3

QUIZ

1. At state 1 in a convergent divergent nozzle with a mass flow rate of air of .02 slugs/sec, the area is 7 in^2. The static pressure and temperature at state 1 are 30 psia and 1200°F. Assuming isentropic flow, find the area of the nozzle where $M = 1.30$.
2. A circular duct is carrying high speed oxygen ($M_1 = .60$) with temperature, $T_1 = 80°F$ and pressure, $P_1 = 30$ psig. Atmospheric pressure is 14 psia. If the pitot tube is inserted in the flow, what would the manometer read? Also calculate the stagnation temperature and the mass flow rate.



QUIZ ANSWERS

1. $T = 1200 + 460$

$T = 1660°R$

$G = .02\ (32.2) = .644$ lb/sec

$A_1 = \frac{7}{144}\ ft^2$

$\gamma_1 = \frac{P_1}{RT_1} = \frac{30(144)}{(53.3)(1660)} = .0488$

$V_1 = \frac{G}{A\gamma} = \frac{.644}{(\frac{7}{144})(.0488)} = 272$ fps

$a_1 = \quad KRgT \quad = \quad 49 \quad 1660 \quad = \quad 40.72(49) = 1996$

$M_1 = \frac{272}{1996} = .1361$

$\frac{T_o}{T} = 1 + M^2\ (\frac{K-1}{2})$

$T_o = 1770\ [1 + (.1361)^2(.2)] = [1.00371](1660)$

$T_o = 1667°R$

At state 2, $M = 1.3$

$\frac{T_o}{T} = 1 + (1.3)^2(.2) = 1.338$

$T_2 = \frac{1667}{1.338} = 1246°R$

also, $(\frac{\gamma_2}{\gamma_1})^K = (\frac{\gamma_2}{\gamma_1})(\frac{T_2}{T_1})$

$$(\frac{\gamma_2}{\gamma_1})^{K-1} = \frac{T_2}{T_1}$$

$$\frac{\gamma_2}{\gamma_1} = (\frac{T_2}{T_1})^{\frac{1}{K-1}}$$

$$\gamma_2 = (.0488)\left(\frac{1.246}{1.660}\right)^{\frac{1}{.4}}$$

$$\gamma_2 = .0238 \text{ lb/ft}^3$$

$$A_2 = 49 \quad 1246 = 35.3(49) = 1730 \text{ fps}$$

$$\frac{V_2}{A_2} = 1.3;\ V_2 = 1.3\ (1730) = 2250 \text{ fps}$$

$$A_2 = \frac{(.644)(144)}{(.0238)(2250)} = 1.729 \text{ in}^2$$

2. $M_1 = .60$

$T_1 = 80°F$

$P_1 = 30$ psig

$P_1 = 30 + 14 = 44$ psia

$R = 48.3$

$K = 1.4$

Manometer reads difference between 14 psia and P_o (stagnation pressure)

a) $$P_o = P\left(1 + \frac{K-1}{2}\ M^2\right)^{\frac{K}{K-1}}$$

$$P_o = 44\ [1 + .2(.36)]^{\frac{1.4}{.4}}$$

$$P_o = 44\ (1.072)^{3.5}$$

$$P_o = 56.1 \text{ psia}$$

$$R_{man.} = (56.1 - 14) = 42.1 \text{ psi}$$

$$R = \frac{(42.1)(144)}{(62.4)(13.55)} = 7.17 \text{ ft}$$

b) $T_o = T\left(1 + \frac{K-1}{2} M^2\right)$

$T_o = 540\ (1 + .072)$

$T_o = 580°R$

c) $\dot{m} = VA\rho$

where

$$\rho = \frac{P}{RT} = \frac{44(144)}{48.3(540)g}$$

$$\rho = 755\ \frac{\text{ft-lb}}{\text{lb}}$$

$$V = .6\ \sqrt{(1.4)(48.3)(32.2)(540)}$$

$V = 650$ fps

$$\dot{m} = .6\ \sqrt{KRgT_1}\ \ \frac{\Pi}{4}\left(\frac{1}{4}\right)\frac{(44)(144)}{RT_1}$$

$\dot{m} = 31.0\ \text{lb}_m/\text{sec}$

MASTER PROBLEM SET KEY

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MASTER RESOURCE MATERIAL LIST

Reference No.	
1	** Fluid Mechanics by V. L. Streeter (5th edition)
2	Handbook (of Tables), Applied Engineering Science
3	* FM-72 Film Loop "Examples of Surface Tension," Encyclopedia Britannica Film Series
4	* FM-47 "Pathlines, Streaklines, Streamlines, and Timelines in Steady Flow"
5	* FM-48 "Pathlines, Streaklines, and Streamlines in Unsteady Flow"
6	* FM-36 "The Venturi Passage"
7	Similitude by V. J. Skoglund
8	Dimensional Analysis and Theory of Modeling by H. L. Langhaar
9	* FM-33 "Stagnation Pressure"
10	* FM-4 "Separated Flow", Part 1
11	* FM-4 "Separated Flow", Part 2
12	Flow of Fluids Through Valves Fittings and Pipes Crane Co., Technical Paper No. 410
13	* FM-8 "The Occurrence of Turbulence"
14	* RFM-134 "Laminar and Turbulent Pipe Flow"
15	* RFM-136 "Structure of Turbulence"

* Film loops may be obtained from: National Committee for Fluid Mechanics Films; Encyclopedia Britannica, Educational Corporation.

** Problems in the problem set of each module (except for module 6) were obtained from this reference.

SAMPLE MODULE

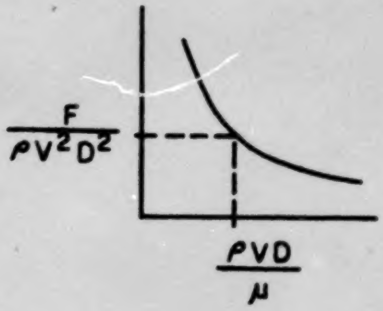
Module Number 6 is used as the sample module. The same format already used in the module appears here except for the addition of the transcript of the lectures that would be given in a lecture-type class. Although the tapes are not used at this point, it should be apparent from the transcribed tapes that an entire course can be set up in such a manner. The following headings would appear in the modules of such a course: tapes and lecture notes, subject matter, incremental objectives, problem index, problem set, problem set answers, resource material, module quiz, and module quiz answers. Again, only the transcript of the tapes and lecture notes from Module 6 is included here since the other headings have appeared previously.

MODULE 6

Similitude is the topic that will be discussed here. There are really two concepts involved in similitude: modeling, and dimensional analysis. These are separate ideas although they are related. First, let's look at the concept of dimensional analysis. Data taken from the laboratory must be arranged in a form suitable for analysis and presentation. For example, suppose you were interested in determining the drag force F on a smooth sphere of diameter D moving through a viscous incompressible fluid (density ρ and viscosity μ) with velocity V. The drag force is some function of these variables

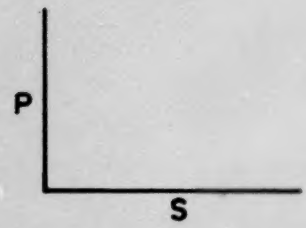
$$F = F(D, \rho, \mu, V)$$

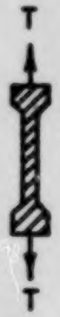
It would be difficult to derive a general analytical equation; therefore, work is done in the laboratory to obtain data which is used in a general analysis. What is meant by general is to apply the results of several experiments to a wide number of applications. If the correct relationship can be obtained for the variables, then any of the variables can be changed and the experimental results should hold for all the changes. Suppose you tried to determine the relationship of the drag force and the above variables by holding three of the variables constant and changing the other. In holding D, μ, ρ constant and varying the velocity V, you would come up with many graphs of F vs V. The same would hold true in changing another variable and holding the others constant. Trying to combine all of these graphs into a correlation would become quite tedious. An alternative procedure is to find groups of variables that are significant rather than relationships between individual variables. This is the purpose of dimensional analysis--to find out what the nondimensional groups of variables are. These nondimensional groups are called Pi (Π) groups. As an example, there is a Pi group known as Π_1, where $\Pi_1 = F/\rho V^2 D^2$ (F = drag force). If you plug the units in here you will see that this is a dimensionless group. Another Pi group, Π_2, takes the form of $\Pi_2 = \rho VD/\mu$. If you plot Π_1 vs Π_2, you will find that all the experimental data will collapse on one simple curve.



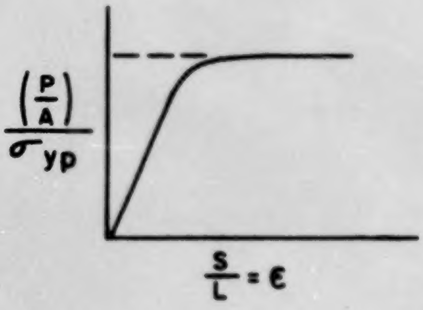
You can see the obvious advantage of plotting data in this manner. (The curve comes from laboratory experimentation.) By knowing what the fluid is, the velocity V, and the diameter D, you can go into the graph and determine the drag force F.

Let's look at a few more examples where dimensional analysis is important. In boundary layer studies the turbulent velocity profile is represented by the equation $u/U_{max} = (y/R)^{L/n}$ where u = velocity, y = distance from wall, and R = distance. Note, both sides of the equation are nondimensional. The same procedure is used here to determine an unknown variable once the other variables are given. In strength of materials, if you wanted to determine the yield strength or modulus of elasticity of a material, you would subject a standard specimen to uniaxial tension. A curve of the load (P) versus axial deformations (δ) would not be very meaningful because the value of the load and of the deformation depends upon the size of the specimen. Yet the size can be eliminated from computations and a stress-strain diagram (indicative of the material itself and not the geometry)





can be made by plotting the force divided by the area [this term divided by the yield point of the material (σ_{yp}) to normalize it] versus strain (ε). Strain represents the deformation (δ) divided by the original gage length of the material (L). Thus $\varepsilon = \frac{\delta}{L}$.



Both of these expressions are nondimensional, and the diagram would be called the normalized stress-strain diagram. It is very useful in design for determining the load carrying capacity of materials.

Since the nondimensional analysis is useful, it is important to be able to derive such Pi (Π) groups or nondimensional groups. The first step in the rational procedure for deriving the groups is to determine the important variables in the problem. This is a very important step and should not be taken lightly. For our purposes here, we will assume that the important variables in the problem (A_1) are known. A general equation might be as follows:

$$A_1 = F\,(A_2, A_3 \ldots A_n)$$

This means that A_1 is a function of A_2, A_3, ... A_n. This expression can be rewritten such that A_1 is included in the function:

$$g\,(A_1, A_2, A_3, \ldots A_n) = 0$$

Once Pi groups are formed, a new relationship (usually obtained from laboratory investigation) will result

$$\Pi_1 = h\,(\Pi_2, \Pi_3 \ldots)$$

where each Pi group is related to the variables by an expression such as

$$\Pi = A_1^{x_1} A_2^{y_2} \ldots .$$

The question is how can the group be formed? How many Pi groups will there be for a given set of variables? The number of Pi groups, n, is equal to the number of variables in the problem, m, minus r, the rank of the dimensional matrix.

$$R = m - r$$

For the function above, the dimensional matrix is formed as follows:

$$\begin{array}{cccc} & A_1 & A_2 & A_3 \\ F & & & \\ L & & & \\ T & & & \end{array}$$

The fundamental dimensions needed to express the variables (A_1, A_2, A_3) are assumed to be F, L, T (F = force, L = length, T = time).

Let's try setting up the matrix for the drag force function previously mentioned where $F = f(D, \rho, \mu, V)$. The columns of the matrix would be the variables of the problem: force, velocity, diameter, viscosity, and density. The fundamental dimensions needed to express the variables are F, L, T.

	force	velocity	diameter	viscosity	density
F					
L					
T					

To complete the matrix, determine the dimensions contained in each variable. For our example

	force	velocity	diameter	viscosity	density
F	1	0	0	1	1
L	0	1	1	-2	-4
T	0	-1	0	1	2

The variable, force, has a force dimension involved; therefore, a one is placed under force in the F row. Velocity and diameter have no force dimension; therefore, a zero is placed in the velocity and diameter columns in the F row. The variable, force, does not have a length dimension so a zero is placed in the force column on the L row. Since velocity $(\frac{\text{distance}}{\text{time}})$ and diameter (length) do have a length dimension, a one is placed as shown in the matrix. The variables, force and diameter, do not have a time dimension, thus zeros appear in these columns on the T row. Since velocity $(\frac{\text{distance}}{\text{time}})$ has a time dimension $(\frac{1}{T})$, a negative one is placed as shown. Once the matrix is complete, the rank of the matrix can be determined. The rank is the smallest nonzero determinate that can be obtained from the matrix. Recall that a determinate can only be formed from a square matrix; therefore, the rank can never be larger than the number of dimensions needed to define the variables in the problem.

Normally (99% of the time) the number of fundamental dimensions needed to express these variables will be equal to the rank of the matrix, r. In this case, this is true since the rank, r = 3. Also, from the given matrix (including viscosity and density) three, three by three determinants can be formed. Two of those might be zero, but one of these is not zero. As long as you find one that is not zero, that is the rank of the matrix. If you found that all the three by three determinants were zero, then you would take two by two determinants. This procedure is continued until you find one that is not zero. Then that will be the rank of the matrix. Usually the rank of the matrix is equal to the number of dimensions needed to express the variables. The number of Pi (Π) groups can be computed as $n = m - r$, where m is the number of variables and r is the rank of the matrix.

Let's look at a Pi group such as:

$$\Pi_1 = A_1^{x_1} A_2^{y_1} A_3^{z_1} A_4$$

This group contains the variables A_1, A_2, A_3, A_4, and exponents x_1, y_1, z_1. The number of variables that will have exponents will equal the number of fundamental dimensions needed to express the problem. Therefore, Π_1 is represented as such:

$$\Pi_1 = A_1^{x_1} A_2^{y_1} A_3^{z_1} A_4$$

A_1, A_2, A_3 are called repeating variables. Note: The number of repeating variables equals the number of fundamental dimensions needed to express all the variables. In our diagram example, the number of repeating variables is three since there are three fundamental dimensions needed (F, L, T). These repeating variables are a subset of the total number of variables. How should they be chosen? The "rule of thumb" is to choose these three such that the following terms are involved: geometric, kinematic, and dynamic. Length and diameter (D) are examples of geometric terms; velocity is a kinematic term; density is a dynamic term (pertains to the inertia of the mass). You should not choose as a repeating variable the variable you want to see one time in the final result. If you choose it as a term that occurs one time in one Pi group, you can then solve explicitly for it in your final expression. Further, the repeating variables should not, by themselves, form a dimensionless group (for obvious reasons).

For example, in determining the relationship for the drag force:

$$F = f(\rho, V, D, \mu)$$

$$g(F, \rho, V, D, \mu) = 0$$

$$n = m - r$$

$$n = 5 - 3 = 2$$

The number of Pi groups is two since the number of variables (m) is five and the rank (r) is three. Therefore, a Π_1 and Π_2 group is formulated as such:

$$\Pi_1 = D^{x_1} V^{y_1} \rho^{z_1} F$$

$$\Pi_2 = D^{x_2} V^{y_2} \rho^{z_2} \mu$$

Notice that the repeating variables are D, V, ρ and that the force (F) is not included as a repeating variable. D represents the geometric term, V represents the kinematic term, and ρ represents the dynamic term. The equations above are to be solved such that Π_1 and Π_2 become dimensionless groups. This will be true provided

$$\Pi_1 = F^0 L^0 T^0$$

$$\Pi_2 = F^0 L^0 T^0$$

The next step is to write the terms (F, ρ, V, D, μ) with respect to the basic dimensions and plug these expressions into the equations of Π_1 and Π_2 and solve the algebraic equations that come from trying to make the Pi terms dimensionless. For the FLT system, the expressions of basic dimensions representing the terms are derived as such:

$$F = \text{force} = F \text{ (FLT system)}$$

$$\rho = \text{density} \rightarrow \frac{\text{mass}}{\text{volume}}$$

$$\text{where } F = ma \rightarrow m \frac{L}{T^2}$$

$$m = \frac{FT^2}{L} \text{ , therefore } \rho \rightarrow \frac{FT^2}{L^4}$$

$$\text{since volume} \rightarrow L^3$$

$$V = \text{velocity} = \frac{\text{distance}}{\text{time}}$$

$$V = \frac{L}{T}$$

$$D = \text{diameter} \rightarrow \text{length} \rightarrow L$$

$$\mu = \tau \frac{dy}{du} \rightarrow \frac{FT}{L^2} \text{ since } \tau = \frac{F}{L^2} \quad , \quad dy \rightarrow L \text{ ,} \quad du \rightarrow \frac{L}{T}$$

Term	FLT system
F	F
ρ	$\frac{FT^2}{L^4}$
V	$\frac{L}{T}$
D	L
μ	$\frac{FT}{L^2}$

These are the basic dimensions for the terms that are used to express the Pi groups. To determine the dimensionless expression for Π_1, apply the equations for Π_1 and substitute the basic dimensions into the equation for the variables as follows:

$$\Pi_1 = F^o\ L^o\ T^o = (L)^{x_1}\ (\frac{L}{T})^{y_1}\ (\frac{FT}{L^4})^{z_1}\ F$$

Solve the equation for the terms F, L, and T. To do this, look at the exponents of each term on both sides of the equation. Take the first basic dimension F

$$F)\quad 0 = z_1 + 1 \qquad \text{solving,} \quad z_1 = -1$$

The exponent of F on the left hand side of the equation is zero. The next exponent of F that appears is on the right side of the equation in the expression

$$(\frac{FT^2}{L^4})^{z_1}$$

The exponent for this is + z_1. And the last of F appears also on the right side in the expression, F. Therefore, the equation for solving F is:

$$0 = z_1 + 1 \quad \text{where } z_1 = -1$$

The following equations are set up for L and T:

$$L)\quad 0 = x_1 + y_1 - 4z_1$$

$$T)\quad 0 = -\ y_1 + 2z_1$$

Using the equation for T, $y_1 = 2z_1$. Substituting this value for y_1 into the equation for L:

$$0 = x_1 + 2z_1 - 4z_1$$

$$x_1 = 2z_1$$

Since $z_1 = -1$ and $y_1 = 2z_1$, then $y_1 = 2(-1) = -2$. Also $x_1 = 2z_1$, therefore $x_1 = 2(-1) = -2$.

By using the above values ($x_1 = -2$, $y_1 = -2$, $z_1 = -1$), the expression for Π_1 can be derived. Substitute the values of x_1, y_1, and z_1 into the previous equation,

$$\Pi_1 = D^{x_1} V^{y_1} \rho^{z_1} F$$

$$\Pi_1 = D^{-2} V^{-2} \rho^{-1} F$$

$$\Pi_1 = \frac{F}{\rho V^2 D^2}$$

This is the dimensionless Pi group that can be used to relate the variables to one another. A simple check to verify the calculations is to substitute the dimensions into the equation and verify that it is dimensionless.

The same procedure is followed to derive the expression for Π_2:

$$\Pi_2 = D^{x_2} V^{y_2} \rho^{z_2} \mu$$

$$\Pi_2 = (L)^{x_2} \left(\frac{L}{T}\right)^{y_2} \left(\frac{FT^2}{L^4}\right)^{z_2} \frac{FT}{L^2}$$

L) $0 = x_2 + y_2 - 4z_2 - 2$

T) $0 = -y_2 + 2z_2 + 1$

F) $0 = z_2 + 1$

Solving the equations,

$z_2 = -1$ (from equation of F)

$0 = -y_2 + 2(-1) + 1$ (from equation of T)

$0 = -y_2 - 2 + 1$

$y_2 = -1$

$0 = x_2 + (-1) - 4(-1) -2$ (from equation of L)

$0 = x_2 - 1 + 4 - 2$

$x_2 = -1$

By using the above values ($x_2 = -1$, $y_2 = -1$, $z_2 = -1$), the expression for Π_2 can be derived. Substitute the values of x_2, y_2, and z_2 into the previous equation,

$$\Pi_2 = D^{x_2} V^{y_2} \rho^{z_2} \mu$$

$$= D^{-1} V^{-1} \rho^{-1} \mu$$

$$\Pi_2 = \frac{\mu}{\rho D V}$$

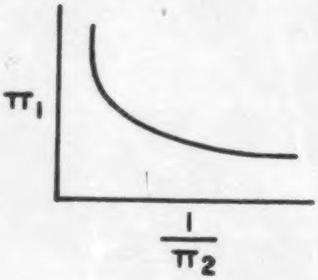
The relationship between Π_1 and Π_2 is that Π_1 is some function of Π_2:

$$\Pi_1 = f\left(\frac{1}{\Pi_2}\right)$$

It is somewhat easier to plot $\frac{1}{\Pi_2}$ rather than Π_2. (This has been determined from previous laboratory work.) Therefore,

$$\frac{F}{\rho V^2 D^2} = F\left(\frac{\rho V D}{\mu}\right)$$

The resulting graph will be as follows:



This graph (universal curve) can be used to determine the unknown variables of a problem when certain other variables are given. This theorem of dimensional analysis is called the Buckingham Pi Theorem.

Modeling--The second part of work done on a model concerns the concept of modeling. The general criteria is called similarity. Modeling and similarity are two words that "go hand in hand", and these two concepts determine whether the prototype and the model will be alike. The model will be worked on and the results will be applied to the prototype in the following paragraphs. An example of a prototype is the actual Boeing 707 aircraft. Extensive studies of models of this aircraft were used in wind tunnels to determine the characteristics of the design. From these model tests, many of the design criteria were formulated.

Similarity is usually broken down into three categories: geometric, kinematic, and dynamic similarity. The first category, geometric similarity, deals with the dimensions of the model and the prototype. All the dimensions of the model have to be scaled by the same ratio as the dimensions of the prototype. If it is an authentic, true scale model, everything is scaled down by the same ratio. Therefore, the ratio of some length of the model (L_m) to some length of the prototype (L_p) is the scale factor (S.F.).

$$\left(\frac{L_m}{L_p}\right) = \text{S.F.}$$

If the integrity of the scale factor is not maintained, then the model will be distorted and will not look like the prototype. The ratio of the model area (A_m) to the prototype area (A_p) is equal to the scale factor squared.

$$\left(\frac{A_m}{A_p}\right) = (\text{S.F.})^2$$

The ratio of the model volume (V_m) to the prototype volume (V_p) is equal to the scale factor cubed.

$$\left(\frac{V_m}{V_p}\right) = (\text{S.F.})^3$$

This relationship can be verified since volume (V) equals to the length (L) cubed. Since

$$\left(\frac{L_m}{L_p}\right) = \text{S.F.} \qquad \text{then } \left(\frac{V_m}{V_p}\right) = (\text{S.F.})^3$$

Usually there is good control over the geometric similarity. An example of one problem related to this in fluid mechanics is defining what a smooth tube is. The wall surface of a pipe might feel smooth to the touch, but under a microscope, the surface may look like a mountain range. In modeling of pipes, smoothness of the wall becomes a factor. Suppose the ratio of the wall boundary roughness (r) to the radius of the pipe (R) is small for a large pipe.

$$\frac{r}{R} = \text{relative roughness}$$

If a model is used which is significantly smaller than the prototype, a problem arises in keeping this same ratio. It might even be impossible to get R in a range where the value of $\frac{r}{R}$ will be geometrically similar with respect to the prototype. This is a problem in maintaining strict geometric similarity.

The second category of similarity is kinematic similarity. Kinematics is related to motion. The terms used to describe motion are velocity and acceleration. For kinematic similarity it must be insured that the velocity ratio for the model and the prototype is the same at corresponding points.

$$\frac{\text{velocity of model at point 2}}{\text{velocity of model at point 2}} = \frac{\text{velocity of prototype at point 1}}{\text{velocity of prototype at point 2}}$$

$$\text{or} \qquad \left(\frac{V_{m_1}}{V_{m_2}}\right) = \left(\frac{V_{p_1}}{V_{p_2}}\right)$$

This insures that the kinematics of the flow (the velocity and acceleration) are in the same ratio.

$$\frac{\text{acceleration of model at point 1}}{\text{acceleration of model at point 2}} = \frac{\text{acceleration of prototype at point 1}}{\text{acceleration of prototype at point 2}}$$

$$\text{or} \qquad \left(\frac{A_{m_1}}{A_{m_2}}\right) = \left(\frac{A_{p_1}}{A_{p_2}}\right)$$

Since velocity and acceleration are vector quantities, the vector nature of these quantities must be considered. This makes it even more complex.

The third category of similarity is dynamic similarity, which is probably the most important of the three. In dynamic similarity, it must be insured that the forces on the model and the prototype are in the same ratio. The ratio of forces at two points on the prototype must be equal to the forces at the same point on the model.

$$\left(\frac{F_1}{F_2}\right)_p = \left(\frac{F_1}{F_2}\right)_m$$

This has to be true for all forces both individually and collectively. Some of the forces found in fluids are: shear (viscous), gravity, inertia, pressure, surface tension, and elastic forces. Using reference parameters;

shear force = viscosity x velocity gradient x area

$$F_s = \mu \times \frac{V\text{, reference velocity}}{L\text{, reference length}} \times L^2$$

$$F_s = \mu VL$$

gravity force = specific weight x volume

$$F_g = \gamma \times L^3$$

$$F_g = \gamma L^3$$

inertia force = mass x acceleration

$$F_I = m \times a$$

where m = mass density (ρ) x volume (L^3)

$$m = \rho L^3$$

and $a = \frac{\text{length}}{(\text{time})^2} = \frac{L}{T^2}$

$$F_I = \rho L^3 \times \frac{L}{T^2} = \frac{\rho L^4}{T^2} \text{ or } \rho L^2 V^2$$

pressure force = pressure drop x area

$$F_p = \Delta P \times L^2$$

$$F_p = \Delta P L^2$$

surface tension force = σ x L

where σ has units of $\frac{\text{force}}{\text{length}}$

This parameter gives the surface tension; therefore, it is necessary to multiply by the reference parameter, length (L).

$$F_{ST} = \sigma L$$

elastic force = compressibility of fluid x area

$$F_e = K \times L^2$$

where K = bulk modules of elasticity

$$F_e = KL^2$$

This force deals with the elasticity of the fluid (compressible fluid--one that changes its volume, density, or specific weight with the flow).

These are the kinds of forces that are seen while working with fluids. They do not usually occur simultaneously. For example, when talking about a gas, there is no surface tension.

For dynamic similarity the force ratios of the model and prototype must be the same. This may be expressed in the following manner.

$$\frac{\Sigma F_m}{\Sigma F_p} = \frac{(F_S + F_g + F_p + F_{ST} + F_e)_m}{(F_S + F_g + F_p + F_{ST} + F_e)_p} = \frac{(F_I)_m}{(F_I)_p}$$

This equation is in terms of Newton's law where the summation of forces (not considering the inertia forces) is equal to the inertia force. In other words, the summation of forces equals to mass times acceleration

(F = ma). These ratios must be the same individually as well as collectively for dynamic similarity.

$$\frac{(F_S)_m}{(F_S)_p} = \frac{(F_g)_m}{(F_g)_p} = \frac{(F_p)_m}{(F_p)_p} = \frac{(F_{ST})_m}{(F_{ST})_p} = \frac{(F_e)_m}{(F_e)_p} = \frac{(F_I)_m}{(F_I)_p}$$

In some problems not all of these forces will occur; therefore, it is easier to work with them in a non-dimensional manner. This can be done by dividing through the equation by one of the force terms. The fact that there is an equation

$$\frac{(F_S + F_g + F_p + F_{ST} + F_e)_m}{(F_S + F_g + F_p + F_{ST} + F_e)_p} = \frac{(F_I)_m}{(F_I)_p}$$

means that all of the forces are not independent. There are six forces, yet one of them will be dependent on the other forces. There is a choice of which force to use as the normalizing force. Usually in fluids, velocity effects are in almost all the forces involved in the study (most of the problems concerned with will be governed by velocity). Therefore, the inertia force is chosen as the normalizing force.

In studying just the model, the relationship obtained by normalizing the numerator and denominator with the respective force, F_I:

$$(\frac{F_S}{F_I}) + (\frac{F_g}{F_I}) + (\frac{F_p}{F_I}) + \frac{F_{ST}}{F_I}) + (\frac{F_e}{F_I}) = 1$$

All of these are non-dimensional terms since force is in the numerator and denominator. Substituting the reference values into these expressions gives certain expressions used in different classes of problems. The above non-dimensional expression has been named in honor of a certain leader in fluid mechanics throughout the centuries. In the first ratio,

$$\frac{F_S}{F_I} = \frac{\mu VL}{\rho L^2V^2} = \frac{\mu}{\rho LV}$$

Since μ is very small, the reciprocal of this expression is usually used instead, $\frac{\rho LV}{\mu}$. It is called the Reynolds number.

$$\text{Reynolds number} = \text{Re} = \frac{\rho LV}{\mu}$$

The Reynolds number is the usual parameter occuring in most flow problems. By looking at the expression, it can be seen that when the inertia forces (viscous forces) become large enough (relatively speaking) the inertia effects start to overshadow viscous effects. When the Reynolds number is sufficiently small, viscous forces predominate and the effects of viscous forces are significant. Therefore, the Reynolds number is one of the most important parameters relating the forces of shear to the forces of inertia. In a problem where the only forces that are important are shear and inertia, dynamic similarity is guaranteed provided the Reynolds number is the same for both the model and the prototype.

For the ratio of gravity forces to inertia forces,

$$\frac{F_g}{F_I} = \frac{\gamma L^3}{\rho L^2V^2} = \frac{\gamma L}{\rho V^2} = \frac{gL}{V^2} \quad \text{since} \quad \frac{\gamma}{\rho} = g$$

Usually the value of this expression is smaller than one, so the reciprocal of the expression is used. This is called the Froude number.

$$\text{Froude number} = \frac{V^2}{gL}$$

Since this expression is related to gravity forces and inertia, it is important in problems involving free surfaces, such as open channel flow.

Maintaining the Froude number the same in the model and the prototype guarantees dynamic similarity for free surface flows.

The ratio of the pressure forces to the inertia forces yields

$$\frac{F_p}{F_I} = \frac{\Delta P L^2}{\rho L^2 V^2} = \frac{\Delta P}{\rho V^2}$$

This expression is called the Euler number or the pressure coefficient. A factor of $\frac{1}{2}$ is usually included in the denominator since ρV^2 is related to kinetic energy. (This is done by convention.)

$$\begin{array}{c}\text{Euler number}\\ \text{(pressure coefficient)}\end{array} = \frac{\Delta P}{\frac{1}{2}\rho V^2}$$

The ratio of surface tension forces to the inertia forces results in the following equation:

$$\frac{F_S}{F_I} = \frac{\sigma L}{\rho L^2 V^2} = \frac{\sigma}{\rho L V^2}$$

Again, the reciprocal is used since σ is a very small number. This expression is called the Weber number.

$$\text{Weber number} = \frac{\rho L V^2}{\sigma}$$

In problems where surface tension is important, dynamic similarity is guaranteed when the Weber number of the model and prototype are the same.

For the ratio of elastic forces to inertia forces,

$$\frac{F_e}{F_I} = \frac{K L^2}{\rho L^2 V^2} = \frac{K}{\rho V^2}$$

however, C (speed of sound) = $\sqrt{\frac{K}{\rho}}$

squaring both sides, $C^2 = \frac{K}{\rho}$

Therefore, $\frac{K}{\rho V^2} = \frac{C^2}{V^2}$ or $\frac{C}{V}$

The reciprocal of this expression   Mach number = $\frac{V}{C}$

is called the Mach number. When the Mach number of a body in the problem is equal to one, then the velocity of that body is equal to the speed of sound. When the Mach number exceeds one, the body is moving at a supersonic speed. In subsonic flows ($M < 1$), the compressibility of the fluid must be considered when the Mach number exceeds approximately 0.4.

In working with the three kinds of similarity, certain parameters must be considered in order to have an accurate dimensional analysis. The scale factor must be adhered to for geometric similarity; velocities and accelerations must be in the same ratio for kinematic similarity; forces must be in the same ratio for dynamic similarity. In working with the forces, a decision has to be made as to which are the important forces. Then these must be related to the model and prototype so that a solution can be obtained for the unknown information.

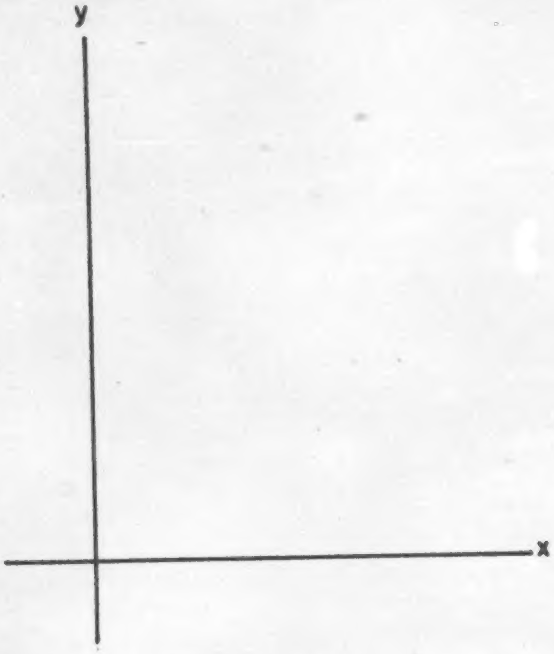
SAMPLE PREREQUISITE EXAM

Instructions

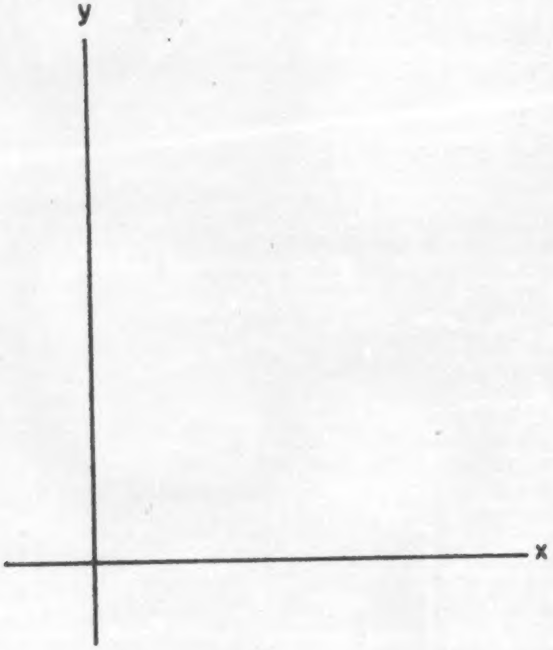
Answer the questions and solve the problems which follow as carefully and accurately as you can. You may use references except in problems which state otherwise to verify information thatyou are familiar with. This is not an open book examination. Guessing or attempting to solve a problem that you are not knowledgeable about will defeat the purpose of the examination which is TO HELP YOU.

When completed, return the examination to your advisor who will review it with you.

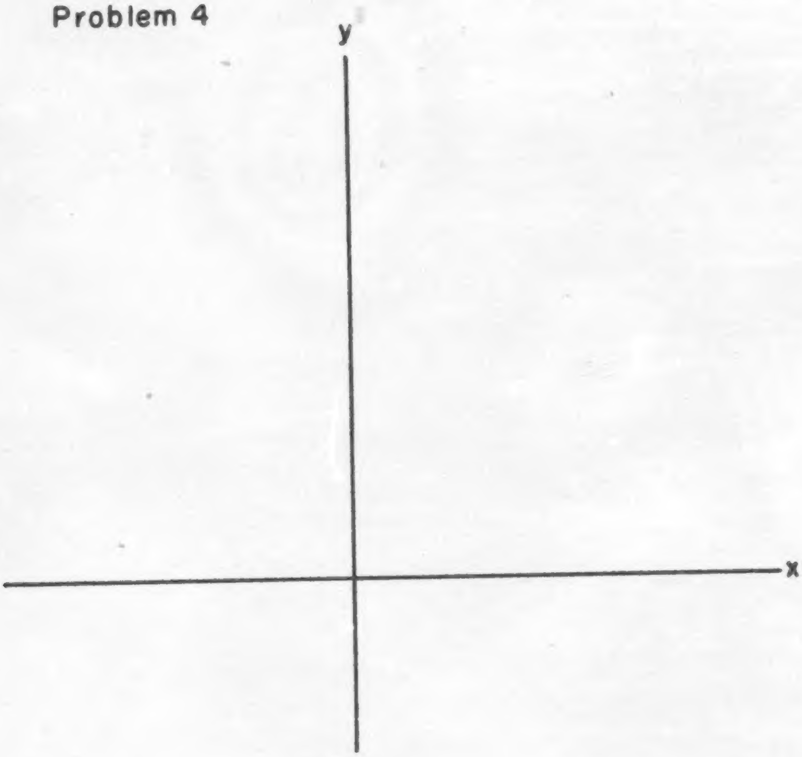
Problem 3a



Problem 3b



Problem 4



I. MATHEMATICS

1. Find the roots of the equation $2x^2 + 3.5x = 4$
2. Compute the values of s, r, and t which satisfy the following set of equations:

$$\begin{aligned} s + 2r + 3t &= 6 \\ 3s \quad\quad + t &= 2 \\ 2s + 4r + t &= 17 \end{aligned}$$

3. Given the equation $y^2 + z^2 - 4x + 4 = 0$
   a) Plot the curve on the xy rectangular coordinate system ($z = 0$
   b) Plot the curve on the yz plane at $x = 0$ and at $x = 2$
   c) Write the resulting equation if the origin is shifted 1 unit in the positive x direction. Note: Plot all curves on attached graph paper
4. Given the equation $y = \frac{1}{4}x^3 - \frac{3}{4}x^2 + 2$

   Compute the critical points (maxima, minima, points of inflection) and use these values to help plot the curve on rectangular coordinates.
5. Give a brief definition of a derivative and show with a simple example a physical interpretation or use.
6. Compute the first derivative of the following functions and evaluate it at the indicated point.
   a) $y = x^4 + 6$ at $x = 3$
   b) $y = x^4 + 6$ at $y = 15$
   c) $u = 0.4 \ln y + 5.5$ at $y = 1$
   d) $v = A \cos 3t$ at $t = \frac{\pi}{4}$

7. Compute the derivative of the following functions:

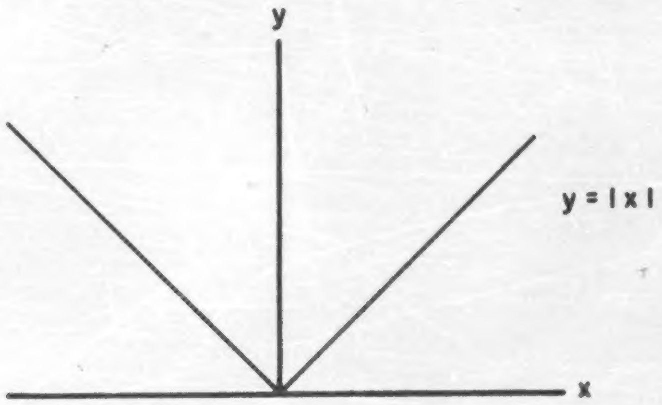
   a) $y = x^2 \cos bx$

   b) $y = \frac{2 + 3x}{1 + x^2}$

8. Examine the function $y = |x|$

   a) Is this function continuous or discontinuous?

   b) Is this function differentiable? If so, evaluate the derivative at the point $x = 0$.



9. Define a partial derivative--how does it differ from the derivatives you compute above?

10. The magnitude of the temperature in a reactor vessel is a function of geometry and time. A typical functional relationship has the form:

$$T = C - [e^{-y}(x^2 + y^2)]$$

where C is a constant

$x = 4t^2$

$y = 2 - t$

Compute the total derivative $\frac{dT}{dt}$ using the chain rule

11. Given the Taylor series expansion

$$f(x) = f(x_o) + (x-x_o)\, f'(x_o) + \frac{(x-x_o)^2}{2!} f''(x_o) + \frac{(x-x_o)^3}{3!} f'''(X_o) + \ldots$$

expand sin x about the point $x_o = \pi/4$. How many terms must one keep in the expansion to evaluate sin $\pi/3$ to three significant decimal places?

12. Show by the fundamental theorem of integral calculus that integration can be related to an area under a curve.

13. Integrate the following functions without a reference:

a) $\frac{bdx}{x + 1}$

b) $12x^3dx$

c) a sin bx dx

d) a cos bx dx

14. Integrate the following functions by parts:

a) x sin x dx

b) ln bx dx

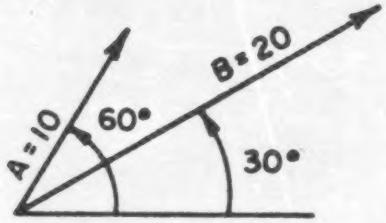
15. A second order differential equation is given by the relationship:

$$\frac{d^2u}{dy^2} = -4$$

Using the boundary conditions u = 0 when $y = \pm y_o$, and u = 10 when y = 0 determine the equation u = f(y)

16. Define a vector and give an example.

17. Vectors $\bar{A}$ and $\bar{B}$ are shown graphically below. Determine the sum $\bar{A} + \bar{B}$.



18. If $\bar{i}$, $\bar{j}$, $\bar{k}$ are the base vectors for a rectangular Cartesian coordisystem and vectors $\bar{A}$ and $\bar{B}$ are defined by:

$$\bar{A} = 3\bar{i} + 2\bar{j} + 7\bar{k}$$

$$\bar{B} = -\bar{i} - \bar{j} + 5\bar{k}$$

a) Determine the scalar product of $\bar{A}$ and $\bar{B}$.

b) Determine the cross product $\bar{A} \times \bar{B}$.

19. A vector associated with a point in space is given by:

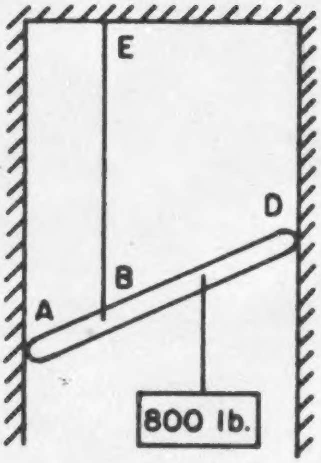
$$\bar{s} = x^2y\bar{i} + xy^2\,\bar{j}$$

Find the component of $\bar{s}$ in the direction of the line $4y - 3x = 0$ at the point (1,1).

20. A particle moves in a circular path with a constant velocity V. Using cylindrical coordinates, compute the acceleration (i.e., derivative of $\bar{V}$ with respect to time).

II. MECHANICS

21. Draw a free body diagram of the bar AD



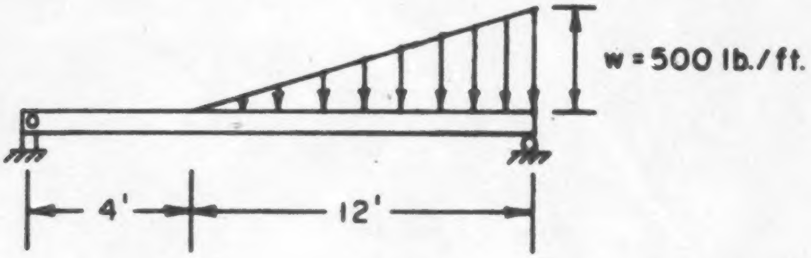
22. A floor beam supports a distributed load as shown

a) Draw a free body diagram of the beam.

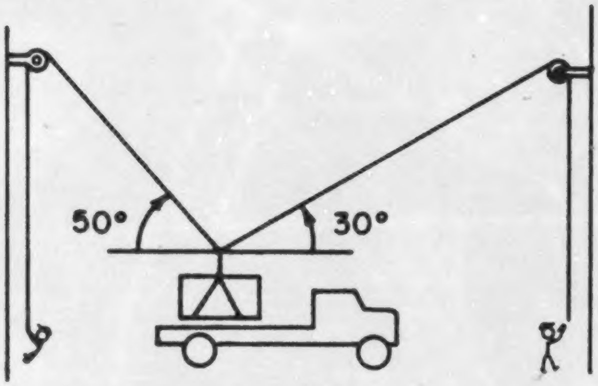
b) Find the equivalent single force which has the same effect on the supports as the original distributed force system.

c) Determine the location of this single force so that the moment is the same as the initial force system.

d) Compute the forces at each support to maintain the beam in equilibrium.

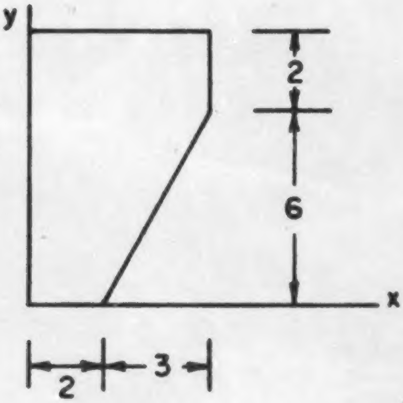


23. Two men are lifting a 200 pound crate from a truck using a system of pulleys. Assuming you wanted to exert the smaller force, what cable would you choose?

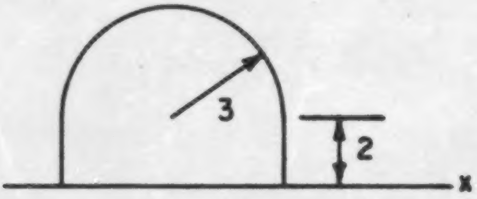


24. The terms centroid, mass center, and center of gravity are sometimes used synonymously. Are they the same? If not, explain the difference.

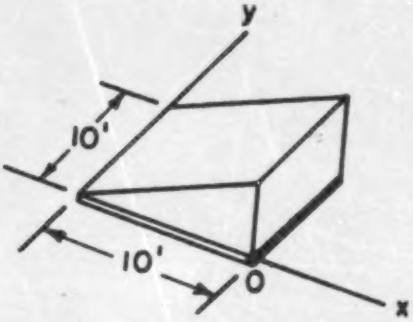
25. For the cross-sectional area shown, compute the x centroidal coordinate.



26. Given the cross-section below, compute the moment of inertia with respect to the x axis.

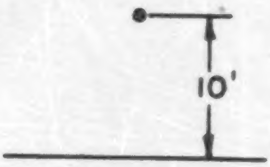


27. The intensity of load per unit area, w, on a horizontal plate varies according to the following equation: $w = 20\,x\ \text{lb/ft}^2$



Compute the magnitude of a single equivalent force which has the same effect as the original loading and locate its position with respect to point O.

28. A solid particle weighing 1/4 pound is dropped from a 10 ft height at a place when the acceleration due to gravity is 32 ft/sec^2



a) What is the maximum potential energy of the particle?

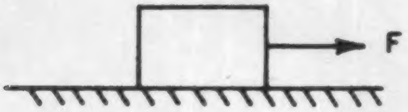
b) Neglecting air resistance, compute the maximum kinetic energy for the particle.

29. The block shown has a mass of 1 kg and moves when a force of 100 Newtons is applied. The kinetic coefficient of friction is 0.4. After 10 seconds:

a) Compute the particle velocity.

b) Compute the particle momentum.

c) Evaluate the work that has been done on the block and the work that has been lost due to friction.



III. THERMODYNAMICS

30. State the first law of thermodynamics. Write the first law in a mathematical form which is applicable to (a) an open system, (b) a closed system.

31. For the following fluids, determine the internal energy:

    a) Steam at 50 psia, 400°F.

    b) Air at 15 psia, 200°F.

32. Determine the specific volume for the fluids in problem 31 in ft^3/lbm.

33. State the second law of thermodynamics and give a brief example of an application.

34. Air is carried through several thermodynamic processes--a to b constant pressure expansion, b to c isentropic expansion and c to a constant temperature compression. Show these processes on the T-S and P-V diagrams and qualitatively describe the heat transfer, work and internal energy changes that will take place.

35. Explain the difference between energy in transition (i.e., heat and work) and intrinsic energy (i.e., internal or kinetic energy).

36. A cylinder fitted with a piston contains 1 lb of air at 500 psia and 180°F. The piston moves outward reducing the pressure to 300 psia while the cylinder volumes doubles. One thousand two hundred fifty (1250) ft lb of work are done by the gas during the process.

    a) Compute the final temperature of the gas in °F.

    b) Find the heat flow in Btu.

    c) Compute the entropy change in $\frac{Btu}{lb}$